CONTENTS

विद्यया च विमुच्यते
SANSKRITI
THE CIVIL SERVICES SCHOOL

SYLLABUS

UNITS		Marks
I	RELATIONS AND FUNCTIONS	8
II	ALGEBRA	10
II	CALCULUS	35
IV	VECTORS AND THREE - DIMENSIONAL GEOMETRY	14
V	LINEAR PROGRAMMING	05
VI	PROBABILITY	08
	Total	80
	Internal Assessment	20

FIRST TERM

March - May

UNIT-VI: PROBABILITY

Probability:

Multiplication theorem on probability. Conditional probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean and variance of random variable. Repeated independent (Bernoulli) trials and Binomial distribution.

UNIT-II: ALGEBRA

Matrices:

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

Determinants:

Determinant of a square matrix (up to 3 x 3 matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle.

Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix

UNIT I. RELATIONS AND FUNCTIONS

Relations and Functions:

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function.

July

Inverse Trigonometric Functions:

Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

UNIT-III: CALCULUS

Continuity and Differentiability:

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit function. Concept of exponential and logarithmic functions and their derivative. Logarithmic differentiation. Derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.

THE CIVIL SERVICES SCHOOL

August

Applications of Derivatives:

Applications of derivatives: rate of change, increasing/decreasing functions, tangents & normals , approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

Integrals:

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, only simple integrals of the type

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{a^2 - x^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{(px+q)dx}{ax^2 + bx + c}$$

$$\int \frac{dx}{\sqrt{ax^2 + bx + c}}, \int \frac{(px+q)dx}{\sqrt{ax^2 + bx + c}}, \int \sqrt{a^2 - x^2}dx, \int \sqrt{x^2 \pm a^2}dx, \int \sqrt{ax^2 + bx + c}dx,$$

$\int (px+q)\sqrt{ax^2 + bx + c}dx$ to be evaluated.

SECOND TERM

October

Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

Applications of the Integrals:

Applications in finding the area under simple curves, especially lines, areas of circles/Parabolas/ellipses (in standard form only), area between the two above said curves (the region should be clearly identifiable).

Differential Equations:

Definition, order and degree, general and particular solutions of a differentiale quation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx} + p(x)y = q(x)$, where p(x) and q(x) are functions of x.

$\frac{dx}{dy} + p(y)x = q(y)$, where p(y) and q(y) are functions of y.

November

UNIT-IV: VECTORS AND THREE-DIMENSIONAL GEOMETRY

Vectors:

Vectors and scalars, magnitude and direction of a vector. Direction cosines/ratios of vectors. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of Scalar (dot) product of vectors, Vector (cross) product of vectors, scalar triple product of vectors.

Three-Dimensional Geometry:

Direction cosines and direction ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii)two planes. (iii) a line and a plane. Distance of a point from a plane.

UNIT-V: LINEAR PROGRAMMING

Linear Programming:

Introduction, definition of related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

SANSKRITI
THE CIVIL SERVICES SCHOOL

Assignment No. 1

Relations and Functions

Note: Q1-7are very short and short answer type questions

1. If the function $f:R\to A$ given by $f(x)=\dfrac{x^2}{x^2+1}$ is surjective, then find A.
2. Let A = $\{2,3,4,5\}$. Define a relation on A which is reflexive and symmetric but not transitive.
3. If $f:R\to R$ and $g:R\to R$ are defined by $f(x)=8x^3;\ g(x)=x^{\frac{1}{3}}$, then find $f\circ g$ & $g\circ f$.
4. If $f(x)=\left(a-x^n\right)^{1/n}$, then find $(f\circ f)(x)$
5. Let $f(x)=x^2-2;\ g(x)=x+2,\ x\in R,$ find $(g\circ f)(1)$
6. If $f(x)=\dfrac{x-1}{x+1}$, find $(fof^{-1})(2)$, assuming that f^{-1} exists.
7. Let f be the greatest integer function and g be the absolute value function, find the value of $(gof)\left(\frac{5}{3}\right)-(fog)\left(\frac{-5}{3}\right)$.
8. Show that $f:R-\{-1\}\to R-\{1\}$ given by $f(x)=\dfrac{x}{x+1}$ is invertible. Also find its inverse.
9. Let $A=\{1,2,3,...,9\}$ and R be the relation on $A\times A$ defined by (a, b) R (c, d) if a + d = b + c for all $(a,b),(c,d)\in A\times A$. Prove that R is an equivalence relation and also obtain the equivalence class [(2,5)].
10. Let N be the set of all natural numbers and R be a relation on $N\times N$, defined as $(a,b)R(c,d)\Leftrightarrow ad=bc$, for all $(a,b),(c,d)\in N\times N$. Show that "R" is an equivalence relation.
11. Let N denote the set of all natural numbers and R be the relation on $N\times N$ defined by $(a,b)R(c,d)$ if $ad(b+c)=bc(a+d)$. Show that R is an equivalence relation.

12. If $f, g: R \to R$ be two functions defined as $f(x)=|x|+x$ and $g(x)=|x|-x$ for all $x \in R$. Then, find fog and gof. Hence, find $fog(-3)$, $fog(5)$ and $gof(-2)$.

13. If $f: R \to R$ is a function defined by $f(x)=x^3+27 \;\forall\; x \in R$. Show that f is bijective and hence find f^{-1}.

14. (i) Let R be the equivalence relation in the set $A=\{0,1,2,3,4,5\}$ given by $R=\{(a,b): 2 \text{ divides } (a-b)\}$. Write the equivalence class $[0]$.

    (ii) Let R be a relation on set of natural numbers N as follows. R= $\{(x,y): x \in N, y \in N \text{ and } 2x+y=24\}$. Find the domain and range of the relation R. Is R an equivalence relation or not?

15. Consider $f: R_+ \to (-9, \infty)$ given by $f(x)=5x^2+6x-9$. Prove that f is invertible with $f^{-1}(y)=\dfrac{\sqrt{54+5y}-3}{5}$

16. If the function $f: R \to R$ be defined by $f(x)=2x-3$ and $g: R \to R$ by $g(x)=x^3+5$ then show that $f \circ g$ is invertible. Also find $(f \circ g)^{-1}(x)$ hence find $(f \circ g)^{-1}(9)$.

Learning Outcomes:

Students will be able to

- define and identify types of relations
- establish given relation to be an equivalence relation
- define and identify types of functions
- understand the composition of functions, find composition of given two or more functions
- define invertible functions
- identify given function as an invertible or non-invertible function
- find inverse of a given function (if it exists).

Assignment No. 2

Inverse Trigonometric Functions

Q 1 – 7 are very short and short answer type questions

Q1. For the principal values, evaluate $\tan^{-1}(-1)+\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$.

Q2. Find the value of $\sin^{-1}\left(\cos\left(\frac{43\pi}{5}\right)\right)$

Q3. Simplify: $\cot^{-1}\frac{1}{\sqrt{x^2-1}}$ for x < -1

Q4. If $4\sin^{-1}x+\cos^{-1}x=\pi$, find the value of x.

Q5. Solve for x: $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$

Q6. If $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$, find x.

Q7. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\sin^{-1}\left(\frac{2b}{1+b^2}\right)=2\tan^{-1}x$, prove that $x=\frac{a+b}{1-ab}$. , $|a|\le 1, |b|\le 1$

Q8. Solve for x: $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$

Q9. If $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$, then find x.

Q10. Prove that if $\frac{1}{2}\le x\le 1$ then $\cos^{-1}x+\cos^{-1}\left(\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right)=\frac{\pi}{3}$

Q11. Simplify each of the following:

a. $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x\ne 0$

b. $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), 0<x<\frac{\pi}{2}$

c. $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right), 0\le x\le 1$

Q12. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\begin{cases}\frac{\pi}{4}+\frac{x}{2}, 0<x<\frac{\pi}{2}\\ \frac{\pi}{4}-\frac{x}{2}, \pi<x<\frac{3\pi}{2}\end{cases}$

Q13. Prove: $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$

Q14. Solve for x: $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

Q15. Solve for x: $\sin^{-1}(1-x)+\sin^{-1}x=\cos^{-1}x$

Learning Outcomes:

Students will be able to

- define the inverse of trigonometric functions and their respective principal value branches.
- find the principal value of the given inverse trigonometric functions.
- state and apply properties of Inverse trigonometric functions to simplify or evaluate a given expression.

SANSKRITI THE CIVIL SERVICES SCHOOL

Assignment No. 3
MATRICES

Questions 1-5 are very short and short answer type questions

1. Find the values of x, y, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$
2. If A is a square matrix, such that $A^2 = A$, then find $(I+A)^2 - 3A$
3. If A= $\left[a_{ij}\right]$ where $a_{ij} = \begin{cases} i+j & if \ \ i \geq j \\ i-j & if \ \ i < j \end{cases}$ then construct a 2×3 matrix A.
4. Prove that the diagonal elements of a skew symmetric matrix are all zero.
5. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$
6. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$
7. Express $\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrix.
8. Find the value of "x" which satisfy $\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$.
9. If $A = \begin{bmatrix} \alpha & 1 \\ 0 & \alpha \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} \alpha^n & n\alpha^{n-1} \\ 0 & \alpha^n \end{bmatrix}$ by principle of Mathematical Induction
10. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$, and $f(x) = x^2 - 4x + 3$, then find $f(A)$
11. An institute conducts classes in two batches I and II and fees for rich and poor children are different. In batch I it has 20 poor and 5 rich children and total monthly collection is Rs 9,000. Whereas in batch II it has 5 poor and 25 rich children and total monthly collection is Rs 26,000. Using matrix method, find monthly fees paid by each child of two types.
12. Find the inverse of each of the following matrices by using elementary transformations:

(i) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Assignment No. 4

Determinants

Questions 1-8 are very short and short answer type questions

1. If for a matrix A , $|A| = 4$, find $|3A|$, where matrix A is of order 2×2
2. A is a non singular matrix of order 3 and $|A| = -5$, find $|adjA|$.
3. If $\left[a_{ij}\right]$ is a matrix of order 3×3, find the value of $a_{11}C_{31} + a_{12}C_{32} + a_{13}C_{33}$, where C_{ij} = the cofactor of a_{ij}.
4. If A is a square matrix of order 3 and $|4A| = k|A|$, find k
5. If $\left[a_{ij}\right]$ is a matrix of order 2×2 such that $|A| = -15$, then find $a_{21}C_{21} + a_{22}C_{22}$, where C_{ij} = the cofactor of a_{ij}.
6. If A is a square matrix of order 3, then find $|adjA|$ if $|A| = 6$.
7. Without expanding the determinant at any stage, prove that $\begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix} = 0$
8. Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, where a, b, c are in A.P.
9. Using matrix method solve the following system of linear equations :

a) $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

b) $5x + 3y + z = 16, 2x + y + 3z = 19, x + 2y + 4z = 25$

10. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB and hence solve the following system of equations: $x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$

SANSKRITI THE CIVIL SERVICES SCHOOL

11. If $A=\begin{bmatrix}3 & 2 & 1\\ 4 & -1 & 2\\ 7 & 3 & -3\end{bmatrix}$, Find A^{-1}, and hence solve $\begin{matrix}3x+4y+7z=14\\ 2x-y+3z=4\\ x+2y-3z=0\end{matrix}$

12. If $A=\begin{bmatrix}5 & 3\\ 12 & 7\end{bmatrix}$, then verify that $A^2-12A-I=0$, where I is a unit matrix of order 2 and hence find A^{-1}.

13. Find the matrix "A" for which $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

14. Area of the triangle with vertices $(-2,4),(2,k)(5,4)$ is 35 square units. Find "k".

15. Using properties of determinants prove that $\begin{vmatrix}y+z & z & y\\ z & z+x & x\\ y & x & x+y\end{vmatrix}=4xyz$.

16. Using properties of determinants prove that

$$\begin{vmatrix}a+b+2c & a & b\\ c & b+c+2a & b\\ c & a & c+a+2b\end{vmatrix}=2(a+b+c)^3.$$

17. If $p\neq 0, q\neq 0$ and $\begin{vmatrix}p & q & p\alpha+q\\ q & r & q\alpha+r\\ p\alpha+q & q\alpha+r & 0\end{vmatrix}=0$ then using the properties of determinants, prove that at least one of the following statements is true (a) p,q,r are in G.P. (b) α is a root of the equation $px^2+2qx+r=0$

18. Using properties of determinants prove that

$$\begin{vmatrix}(b+c)^2 & a^2 & a^2\\ b^2 & (c+a)^2 & b^2\\ c^2 & c^2 & (a+b)^2\end{vmatrix}=2abc(a+b+c)^3$$

19. Using properties of determinants prove that $\begin{vmatrix}a & a+b & a+b+c\\ 2a & 3a+2b & 4a+3b+2c\\ 3a & 6a+3b & 10a+6b+3c\end{vmatrix}=a^3$.

SANSKRITI THE CIVIL SERVICES SCHOOL

20. Using properties of determinants prove that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & bc & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

Learning Outcomes:

Students will be able to

- form matrices of different order, define types of matrices.
- Add, subtract and equate matrices, multiply a matrix by a scalar and express a matrix as a sum of symmetric and skew symmetric matrices
- Multiply two matrices, properties of multiplication
- Find the inverse of a matrix by elementary operations
- define determinant of a matrix and find the determinant of square matrix of order 2.
- find minors, cofactors and adjoint of a square matrix
- to find the inverse of a matrix using its adjoint and determinant
- To evaluate determinant of a matrix using properties
- Solving system of linear equations

SANSKRITI THE CIVIL SERVICES SCHOOL

Assignment No. 5

Continuity and Differentiability

Questions 1 – 10 are very short and short answer type questions

1. If $f(1)=4,\ f'(1)=2$, find the value of derivative of $\log f\left(e^x\right)$ w. r. t. "x" at $x=0$

2. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right), x<0, 0<6^x<1$

3. If $y=e^{x+e^{x+e^{x+e^{x+......}}}}$, Prove that $\frac{dy}{dx}=\frac{y}{1-y}$.

4. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to "x", $|x|<1$.

5. State the points of discontinuity for the function $f(x)=[x]$ in $-3<x<3$

6. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to "x".

7. Differentiate $\tan^{-1}\left(\frac{\sqrt{a}-\sqrt{x}}{1+\sqrt{a}\sqrt{x}}\right)$ w.r.t x.

8. Differentiate $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ with respect to $\tan^{-1}x$, $0<x<\pi$

9. If f is a differentiable function at x = 1 such that $f(1)=5, f'(1)=\frac{1}{5}$ & $g=f^{-1}$,

   then find $g'(5)$

10. If $y=\tan^{-1}\frac{5x}{1-6x^2}, \frac{-1}{\sqrt{6}}<x<\frac{1}{\sqrt{6}}$, then prove that $\frac{dy}{dx}=\frac{2}{1+4x^2}+\frac{3}{1+9x^2}$

11. For what value of "k" the function $f(x)=\begin{cases}\frac{\sqrt{5x+2}-\sqrt{4x+4}}{x-2}; & x\neq 2\\ k & x=2\end{cases}$ is continuous

    at $x=2$.

12. Determine the value of "a", "b" and "c" if the following function is continuous at x=0 $f(x)=\begin{cases} \dfrac{\sin(a+1)x+\sin x}{x}; & \text{when } x \prec 0 \\ c & \text{when } x=0 \\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{\frac{3}{2}}} & \text{when } x \succ 0 \end{cases}$

13. Discuss the derivability of the function $f(x)=\begin{cases} x-1, & x<2 \\ 2x-3, & x \geq 2 \end{cases}$ at $x=2$

14. Find "a" and "b", if the function given by $f(x)=\begin{cases} x^2, & x \leq 1 \\ ax+b, & x>1 \end{cases}$ is differentiable at $x=1$

15. Discuss the continuity of the function $f(x)=|x-1|-|x-2|$.

16. Verify Rolle's theorem for the function $f(x)=e^x(\sin x-\cos x)$ on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

17. If $y=x^{\sin x}+\sin^{-1}\sqrt{x}$, find $\dfrac{dy}{dx}$.

18. If $x^p.y^q=(x+y)^{p+q}$, prove that $\dfrac{dy}{dx}=\dfrac{y}{x}$.

19. If $y=\sin(m\sin^{-1}x)$, prove that $(1-x^2)y_2-xy_1+m^2y=0$.

20. If $x=\sin t,\ y=\sin pt$, prove that $(1-x^2)y_2-xy_1+p^2y=0$.

21. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, Show that $\dfrac{dy}{dx}=\sqrt{\dfrac{1-y^2}{1-x^2}}$

22. If $x=a\left(\cos\theta+\log\tan\left(\frac{\theta}{2}\right)\right)$ and $y=a\sin\theta$ find $\dfrac{d^2y}{dx^2}$ at $\theta=\dfrac{\pi}{4}$

23. If $x=a(1-\cos^3\theta)$ and $y=a(\sin^3\theta)$. Find $\dfrac{d^2y}{dx^2}$ at $\theta=\dfrac{\pi}{6}$

24. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right)$ w.r.t. $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$ where $x\in\left(\dfrac{1}{\sqrt{2}},1\right)$

25. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

Learning Outcomes:

Students will be able to

- discuss continuity of a function at a point of domain
- discuss continuity of a function over the domain of the function
- discuss differentiability of a function at a point of domain
- find derivative of composition of two or more functions, using chain rule
- differentiate inverse trigonometric functions
- differentiate implicit functions
- understand the nature of exponential and logarithmic functions and their derivatives
- use logarithm to differentiate given function
- find derivative of parametric functions
- find second order derivative of a given function
- state and verify Rolle's Mean value theorem, appreciate its geometric and physical significance
- state and verify Lagrange's Mean value theorem, appreciate its geometric and physical significance

विद्या SANSKRITI THE CIVIL SERVICES SCHOOL

Application of Derivatives

1. Water is dripping out from a conical funnel at a uniform rate of 4 cu-cm/sec. When the slant height is 3 cm, find the rate of decrease of slant height of the water cone, given the vertical angle of the funnel is 120˚.
2. The total cost C(x) associated with production of x units is given by $C(x) = 0.0005x^3 - 0.002x^2 + 30x + 3000$. Find the marginal cost when 3 units are produced.
3. A balloon which always remains spherical, has a variable diameter 3 (2x + 5). Determine the rate of change of volume w.r.t x.
4. Find the equation of the tangent to the curve $x = \theta + sin\theta$, $y = 1 + cos\theta$ at $\theta = \pi/4$
5. Find equation of all lines of slope 0 and that are tangent to the curve

$$y = 1/(x^2 - 2x + 3)$$

6. Find equation of the tangent to the curve $y = \sqrt{(3x-2)}$ which is parallel to the line $y = 4x - 5$
7. Find the intervals in which the following functions are strictly increasing or strictly decreasing:

a) $f(x) = 10 - 6x - 2x^2$

b) $f(x) = 2x^3 - 12x^2 + 18x + 15$

c) $f(x) = 5 + 36x + 3x^2 - 2x^3$

d) $f(x) = 5x^3 - 15x^2 - 120x + 3$

e) $f(x) = -2x^3 - 9x^2 - 12x + 1$

f) $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

g) $f(x) = \log(1+x) - \frac{x}{1+x}$

8. *Find all the points of local maxima and minima of the following functions*:

a) $f(x) = x^3 - 3x$

b) $f(x) = x^3 (x-1)^2$

c) $f(x) = (x-1)(x+2)^2$

d) $f(x) = (x-1)^3 (x-1)^2$

e) $f(x) = \sin x - cosx$, $0 < x < 2\pi$

f) $f(x) = \sin x + \cos x$, $0 < x < \pi/2$

g) $f(x) = x^4 - 62x^2 + 120x + 9$

h) $f(x) = x^3 - 6x^2 + 9x + 15$

SANSKRITI THE CIVIL SERVICES SCHOOL

9. Find the absolute maximum and minimum values of the following functions in the given intervals:

a) $f(x) = 4x - \frac{x^2}{2}$ in [-2, 4.5]

b) $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ in [0,3]

c) $f(x) = cos^2x + sinx$, $x \in [0, \pi/2]$

d) $f(x) = (x-2)\sqrt{(x-1)}$ in [1, 9]

10. Show that of all rectangles with given perimeter, the square has largest area.
11. Show that of all rectangles of given area, the square has the least perimeter.
12. Show that of all rectangles inscribed in a given circle, the square has maximum area.
13. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius 'a' is a square. Also find the side of the square.
14. If the sum of the lengths of the hypotenuse and side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.
15. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.
16. Prove that the area of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.
17. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.
18. A figure consists of a semi-circle with a rectangle on its diameter. Given the perimeter of the figure, find the dimensions in order that the area may be maximum.
19. Find the volume of the largest cylinder that can be inscribed in a sphere of radius 'r' cm.
20. Show that a cylinder of given volume which is open at the top, has maximum total surface area, when the height of the cylinder is equal to the radius of the base.
21. Show that the height of the closed cylinder of given surface area and maximum volume is equal to the diameter of the base.
22. Show that the height of a cylinder which is open at the top having a given surface area and greatest volume is equal to the radius of its base.
23. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius 'a' is $2a/\sqrt{3}$.
24. An open box with a square base is to be made of a given quantity of cardboard of area c^2 sq units. Show that the maximum volume of the box is $c^3/(6\sqrt{3})$ cu. units.

SANSKRITI THE CIVIL SERVICES SCHOOL

Assignment No. 6
Application of Derivatives

1. Water is dripping out from a conical funnel of semi vertical angle $\frac{\pi}{4}$ at a uniform speed of 2 $cm^3/\sec$ through a tiny hole at the vertex of the bottom. When the slant height of water is 4 cm, find the rate of decrease of slant height of the water.
2. A man is moving away from a tower 49.6 m high at the rate of 2 m/s. Find the rate at which the angle of elevation of the top of the tower is changing, when he is at a distance of 36 m from the foot of the tower. Assume that the eye level of the man is 1.6 m from the ground.
3. Evaluate following up to three decimal places using differentiation:
$\sqrt{25.2}$, $\sqrt[3]{29}$, $\sqrt{0.037}$
4. Find the intervals in which the function $f(x)=\log(1+x)-\frac{2x}{2+x}$ increasing or decreasing.
5. Find the intervals in which the function $f(x)=(x+1)^3(x-3)^3$ is increasing or decreasing. Also find the points at which the function has local maxima, local minima and the point of inflexion.
6. Find all the points of local maximum and minimum and the corresponding maximum and minimum values of the following function $\frac{3}{4}x^4-8x^3+\frac{45}{2}x^2+105$.
7. Find the point on the curve $y^2=4x$ which is nearest to the point $(2,-8)$
8. Find the equation of the tangent to the curve $y=(x^3-1)(x-2)$ at the points where the curve cuts the x –axis.
9. Find the intervals in which the function $f(x)=2x^3-9x^2+12x+15$ is increasing and decreasing.
10. Separate $\left[0,\frac{\pi}{2}\right]$ into sub intervals in which $f(x)=\sin^4 x+\cos^4 x$ is increasing or decreasing.

11. Find the points of local maxima and local minima and also the local maximum and local minimum values of the following functions: $(i)\ f(x)=2\cos x+x, x\in(0,\pi)$

$(ii)\ f(x)=2\sin x-x, x\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

12. Find the equation of the tangent and normal to the curve $x=1-\cos\theta;\ \ y=\theta-\sin\theta$ at $\theta=\frac{\pi}{4}$

13. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of cone.

14. An open box with a square base is to be made of given iron sheet of area 27 sq. m. Show that the maximum volume of the box is 13.5 cu. m.

15. Find the coordinates of a point on the parabola $y=x^2+7x+2$ which is closest to the line $y=3x-3$

16. Find the equation of the normal to the curve $2y=x^2$, which passes through (2,1).

Learning Outcomes:

Students will be able to

- find slope and equation of tangent and normal to a given function at a given point.
- use derivatives to find rate of change
- determine the nature of a given function as an increasing or decreasing function using derivatives
- find intervals over which the function is increasing/decreasing
- find the points of local maxima, local minima, local maximum and local minimum value of a given function (if any)
- find the absolute maximum and absolute minimum value of a given function
- apply the concept of maxima and minima to solve word problems
- use derivative of a function to find approximate values

Indefinite integral

Some generalized results of the Method of substitution:

$\int f(x)\times f'(x)dx=\int tdt$, where $f(x)=t$, $\int f(g(x))\times g'(x)dx=\int f(t)dt$, where $g(x)=t$

Formula: $\int \frac{f'(x)}{f(x)}dx=\log|f(x)|+c$

This formula may be derived by using the method of substitution.

Results based on the above formula:

1. $\int \tan x dx=\log|\sec x|+c$ 2. $\int \cot x dx=\log|\sin x|+c$

3. $\int \sec x dx=\log|\sec x+\tan x|+c=\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|+c$

4. $\int cosec\, x dx=\log|cosec\, x-\cot x|+c=\log\left|\tan\frac{x}{2}\right|+c$

Direct application of the above formula in finding the following integrals:

1. $\int \frac{2x}{1+x^2}dx$ 2. $\int \frac{x}{9-4x^2}dx$ 3. $\int \frac{e^{2x}-1}{e^{2x}+1}dx$ 4. $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$ 5. $\int \frac{\sin x}{1+\cos x}dx$ 6. $\int \frac{2\cos x-3\sin x}{6\cos x+4\sin x}dx$

Formula: If $\int f(x)dx=g(x)+c$, then $\int f(ax+b)dx=\frac{1}{a}g(ax+b)+c$

This formula may be derived either by direct differentiation or by using the method of substitution.

Direct application of the above formula in finding the following integrals:

7. $\int \sec^2(7-4x)dx$ 8. $\int \tan^2(2x-3)dx$ 9. $\int \sqrt{ax+b}dx$ 10. $\int e^{2x+3}dx$ 11. $\int \sec 2x dx$

12. $\int \sin(3x-1)dx$

SANSKRITI THE CIVIL SERVICES SCHOOL

The application of method of substitution in finding some basic integrals:

13. $\int \frac{(\log x)^2}{x} dx$ 14. $\int \frac{1}{x + x\log x} dx$ 15. $\int x\sqrt{1+2x^2} dx$ 16. $\int \left(x^3 - 1\right)^{\frac{1}{3}} x^5 dx$ 17. $\int \frac{x}{e^{x^2}} dx$

18. $\int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8} dx$

Integrals of the type: $\int \frac{a\sin x + b\cos x}{c\sin x + d\cos x} dx$

19. $\int \frac{2\sin x + 3\cos x}{5\sin x + 4\cos x} dx$ 20. $\int \frac{1}{1+\cot x} dx$ 21. $\int \frac{1}{1-\tan x} dx$

Integration using trigonometric identities:

22. $\int \sin^2 x dx$ 23. $\int \cos^2 x dx$ 24. $\int \sin^3 x dx$ 25. $\int \cos^3 x dx$ 26. $\int \sin^2(2x+5) dx$

27. $\int \sin^3(2x+1) dx$ 28. $\int \sin 3x \cos 4x dx$ 29. $\int \sin 4x \sin 8x dx$ 30. $\int \cos 2x \cos 4x \cos 6x dx$

31. $\int \sin^4 x dx$ 32. $\int \cos^4(2x) dx$ 33. $\int tan^4 x dx$ 34. $\int \frac{1}{\sin x \cos^3 x} dx$ 35. $\int \sin^2 x \cos^2 x dx$

36. $\int \sin^3 x \cos^3 x dx$

Problems which require multiplication and division by $\sin(a \pm b)$ *or by* $\cos(a \pm b)$

37. $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$ 38. $\int \frac{1}{\sin(x+a)\cos(x+b)} dx$

Integrals of the type: $\int \sin^m x \cos^n x dx$

If m is odd, then put cosx = t and if n is odd, then put sinx = t. If both are odd, then put either of them = t. If both are even, then use suitable trigonometric identities.

39. $\int \sin^3 x \cos^2 x dx$ 40. $\int \sin^4 x \cos^4 x dx$ 41. $\int \sin^5 x dx$

Integrals of the types: $\int \frac{1}{x^2 - a^2} dx$, $\int \frac{1}{a^2 - x^2} dx$, $\int \frac{1}{x^2 + a^2} dx$, $\int \frac{1}{\sqrt{x^2 - a^2}} dx$, $\int \frac{1}{\sqrt{a^2 - x^2}} dx$,

$\int \frac{1}{\sqrt{x^2 + a^2}} dx$

42. $\int \frac{3x^2}{x^6+1}dx$ 43. $\int \frac{1}{\sqrt{1+4x^2}}dx$ 44. $\int \frac{1}{\sqrt{(2-x)^2+1}}dx$ 45. $\int \frac{1}{\sqrt{9-25x^2}}dx$

46. $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}dx$ 47. $\int \frac{\cos x}{\sqrt{\sin^2 x-2\sin x-3}}dx$

Integrals of the types: $\int \frac{1}{ax^2+bx+c}dx,\ \int \frac{px+q}{ax^2+bx+c}dx,\ \int \frac{1}{\sqrt{ax^2+bx+c}}dx,\ \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$

48. $\int \frac{1}{x^2-2x-5}dx$ 49. $\int \frac{1}{9x^2+6x+5}dx$ 50. $\int \frac{5x-2}{3x^2+2x+1}dx$ 51. $\int \frac{x+3}{x^2-2x-5}dx$

52. $\int \frac{1}{\sqrt{x^2+2x+2}}dx$ 53. $\int \frac{1}{\sqrt{7-6x-x^2}}dx$ 54. $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$ 55. $\int \frac{x+2}{\sqrt{4x-x^2}}dx$

56. $\int \frac{2\sin 2\phi-\cos\phi}{6-\cos^2\phi-4\sin\phi}d\phi$

Integrals of the type: $\int \frac{\sin x \pm \cos x}{f(\sin 2x)}dx$

If the numerator is sinx + cosx, then put sinx – cosx = t. If the numerator is sinx - cosx, then put sinx+cosx = t.

57. $\int \frac{\cos x-\sin x}{1+\sin 2x}dx$ 58. $\int \frac{\cos x+\sin x}{\sqrt{\sin 2x}}dx$ 59. $\int \frac{\cos x+\sin x}{9+16\sin 2x}dx$

Integrals of the type: $\int \frac{1}{x^{\frac{1}{a}}+x^{\frac{1}{b}}}dx$

Put $x = t^{\text{the lcm of a and b}}$

60. $\int \frac{1}{x-\sqrt{x}}dx$ 61. $\int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx$

Integration using partial fractions:

Integrals of the type: $\int \frac{f(x)}{(x-a)(x-b)\ldots}dx$ ***(Denominator has non repeating linear factors)***

62. $\int \frac{x}{(x+1)(x+2)}dx$ 63. $\int \frac{1}{x^2-9}dx$ 64. $\int \frac{3x-1}{(x-1)(x-2)(x-3)}dx$ 65. $\int \frac{1-x^2}{x(1-2x)}dx$

66. $\int \frac{2x-3}{(x^2-1)(2x+3)}dx$ 67. $\int \frac{x^3+x+1}{x^2-1}dx$

Integrals of the type: $\int \frac{f(x)}{(x-a)^m(x-b)...}dx$ ***(Denominator has repeating linear factors and/or non repeating linear factors)***

68. $\int \frac{x}{(x-1)^2(x+2)}dx$ 69. $\int \frac{3x-1}{(x+2)^2}dx$ 70. $\int \frac{3x-1}{(x-1)^3(x-2)}dx$

Integrals of the type: $\int \frac{f(x)}{(ax^2+bx+c)(x-d)^m(x-e)...}dx$ ***(Denominator has a quadratic factor which is not resolvable into linear factors and/or has repeating linear factors and/or non repeating linear factors)***

71. $\int \frac{2}{(1-x)(1+x^2)}dx$ 72. $\int \frac{x}{(x-1)(1+x^2)}dx$ 73. $\int \frac{1}{x^4-1}dx$ 74. $\int \frac{2x^2+1}{x^2(4+x^2)}dx$

Integrals of the type: $\int \frac{1}{x(x^n \pm 1)}dx$

75. $\int \frac{1}{x(x^n+1)}dx$ 76. $\int \frac{1}{x(x^4-1)}dx$

Integrals of the type: $\int \frac{(x^2+a)(x^2+b)...}{(x^2+c)(x^2+d)...}dx$

77. $\int \frac{(x^2+1)}{(x^2+2)(2x^2+1)}dx$ 78. $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx$

SANSKRITI THE CIVIL SERVICES SCHOOL

Some indirect problems of partial fractions:

79. $\int \frac{2x}{(x^2+1)(x^2+3)}dx$ 80. $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$ 81. $\int \frac{\sin x}{\sin 4x}dx$ 82. $\int \frac{\tan x+\tan^3 x}{1+\tan^3 x}dx$

83. $\int \frac{1}{\sin x+\sin 2x}dx$

Integration by parts:

84. $\int x\sin x dx$ 85. $\int x^2 e^x dx$ 86. $\int x^2 \log x dx$ 87. $\int x\sin^{-1} x dx$

88. $\int x\tan^{-1} x dx$ 89. $\int \sin^{-1} x dx$ 90. $\int (\sin^{-1} x)^2 dx$ 91. $\int \log x dx$ 92. $\int (\log x)^2 dx$ 93. $\int x(\log x)^2 dx$

94. $\int tan^{-1} x dx$ 95. $\int \sec^3 x dx$ 96. $\int co\sec^3 x dx$

Integrals based on the result: $\int e^x (f(x)+f'(x))dx = e^x f(x)+c$

97. $\int e^x(\sin x+\cos x)dx$ 98. $\int \frac{xe^x}{(1+x)^2}dx$ 99. $\int e^x \frac{(1+\sin x)}{(1+\cos x)}dx$ 100. $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$

101. $\int \frac{(x-3)e^x}{(x-1)^3}dx$ 102. $\int \frac{(x^2+1)e^x}{(1+x)^2}dx$ 103. $\int \left\{\frac{1}{\log x}-\frac{1}{(\log x)^2}\right\}dx$

104. $\int \left\{\log(\log x)+\frac{1}{(\log x)^2}\right\}dx$ 105. $\int \left(\log x+\frac{1}{x^2}\right)e^x dx$

Integrals of the types: $\int e^{ax}\sin bx dx$, $\int e^{ax}\cos bx dx$

106. $\int e^{2x}\sin x dx$ 107. $\int e^{-x}\cos x dx$ 108. $\int e^x \sin^2 x dx$ 109. $\int e^{2x}\cos^2 x dx$

Integrals of the types: $\int \sqrt{a^2-x^2}dx$, $\int \sqrt{a^2+x^2}dx$, $\int \sqrt{x^2-a^2}dx$, $\int \sqrt{ax^2+bx+c}dx$,

$\int (px+q)\sqrt{ax^2+bx+c}dx$

110. $\int \sqrt{4-x^2}dx$ 111. $\int \sqrt{1-4x^2}dx$ 112. $\int \sqrt{(x+2)^2-9}dx$ 113. $\int \sqrt{x^2+4x+6}dx$

114. $\int \sqrt{1+3x-x^2}dx$ 115. $\int (x+3)\sqrt{3-4x-x^2}dx$

Integrals of the types: $\int \frac{1}{a+b\sin x}dx$, $\int \frac{1}{a+b\cos x}dx$, $\int \frac{1}{a\sin x+b\cos x}dx$, $\int \frac{1}{a\sin x+b\cos x+c}dx$

116. $\int \frac{1}{1-2\sin x}dx$ 117. $\int \frac{1}{5-4\cos x}dx$ 118. $\int \frac{1}{3\sin x+\cos x}dx$ 119. $\int \frac{1}{\sin x+\cos x+2}dx$

Some specific Integrals:

120. $\int \frac{x^2+1}{x^4+1}dx$ 121. $\int \frac{x^2-1}{x^4+1}dx$ 122. $\int \frac{x^2}{x^4+1}dx$ 123. $\int \frac{1}{x^4+1}dx$ 124. $\int \frac{1}{x^4+3x^2+1}dx$

125. $\int \frac{x^2+1}{x^4+7x^2+1}dx$ 126. $\int \sqrt{\tan x}dx$, 127. $\int \sqrt{\cot x}dx$

Integrals of the types: $\int \frac{1}{a\sin^2 x+b\cos^2 x}dx$, $\int \frac{1}{a+b\cos^2 x}dx$, $\int \frac{1}{a+b\sin^2 x}dx$, $\int \frac{1}{(a\sin x+b\cos x)^2}dx$, $\int \frac{1}{a\sin^2 x+b\cos^2 x+c}dx$

128. $\int \frac{1}{3+2\cos^2 x}dx$ 129. $\int \frac{1}{1+3\sin^2 x}dx$ 130. $\int \frac{1}{4\sin^2 x+5\cos^2 x}dx$ 131. $\int \frac{\cos x}{\cos 3x}dx$

132. $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$ 133. $\int \frac{1}{(2\sin x+\cos x)(\sin x-2\cos x)}dx$ 134. $\int \frac{1}{2-3\cos 2x}dx$

135. $\int \frac{1}{(2\sin x+3\cos x)^2}dx$

Miscellaneous problems which require some specific steps in order to reduce them to some known form:

136. $\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx$ 137. $\int \sqrt{\frac{a-x}{a+x}}dx$ 138. $\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}}dx$ 139. $\int \sqrt{\sec x-1}dx$ 140. $\int \frac{1}{\sin x+\sec x}dx$

141. $\int \sqrt{\frac{x}{a^3-x^3}}dx$ 142. $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}dx$ 143. $\int \frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}dx$

144. $\int \frac{\sqrt{x^2+1}\{\log(x^2+1)-2\log x\}}{x^4}dx$ 145. $\int \left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

SANSKRITI THE CIVIL SERVICES SCHOOL

Assignment No. 7(a)

Indefinite Integrals-1

(Method of Substitution, Trigonometric Identities, Special Integrals)

Q1-10 are very short and short answer type questions

1. Evaluate: $\dfrac{x-2}{\sqrt{x^2+1}}$
2. Evaluate: $\int x^{1/2}(1+x^{3/2})\, dx$
3. Write a value of $\int \dfrac{1}{\sqrt{3}\sin x+\cos x}\, dx$
4. Write a value of $\int \dfrac{1-\tan x}{x+\log\cos x}\, dx$
5. If $f'(x)=\dfrac{4}{x^2}$ and $f(1)=6$, find $f(2)$
6. Write a value of $\int \left(\cos(\log x)+\sin(\log x)\right) dx$
7. If $\int \dfrac{2^x}{\sqrt{1-4^x}}\, dx = k\sin^{-1}(2^x)+c$, then what is the value of k?
8. Find a value of $\int \dfrac{1}{9x^2-16}\, dx$
9. Evaluate $\int \dfrac{x}{e^{x^2}}\, dx$
10. Evaluate: $\int \dfrac{dx}{e^x+e^{-x}}$
11. $\int \dfrac{1}{(\sin x-2\cos x)(2\sin x+\cos x)}\, dx$
12. Evaluate: $\int \dfrac{1}{\sqrt{\sin^3 x(\sin x+2\cos x)}}\, dx$
13. Evaluate: $\int \operatorname{cosec}^8 x\, dx$
14. Evaluate: $\int 5^{5^{5^x}}.5^{5^x}.5^x\, dx$
15. Evaluate: $\int \dfrac{x^2}{\sqrt{x^6-a^6}}\, dx$
16. Evaluate: $\int \dfrac{x^3+x^2+2x+1}{x^2-x+1}\, dx$
17. Evaluate: $\int \sqrt{\dfrac{3-x}{x-2}}\, dx$
18. Evaluate: $\int \dfrac{e^x}{2e^{2x}+3e^x+1}\, dx$
19. Evaluate: $\int \dfrac{6x+5}{\sqrt{6+x-2x^2}}\, dx$
20. Evaluate: $\int \sqrt{1+\operatorname{cosec} x}\, dx$

Find the integral of the following w. r. to x

21. $\dfrac{1}{(3\sin x+\cos x)^2}$

22. $\dfrac{1}{x\log x\log(\log x)}$

23. $\sqrt{\tan\theta}+\sqrt{\cot\theta}$

24. $\dfrac{1}{\sin^2 x+\sin 2x}$

25. $\dfrac{\log x^2}{x}$

26. $\sqrt{\sec x-1}$

27. $\dfrac{1}{5+7\cos x+\sin x}$

28. $\dfrac{x^2+9}{x^4+81}$

29. $\dfrac{1}{(a^2\sin^2 x+b^2\cos^2 x)^2}$

Assignment No. 7(b)

Indefinite Integrals-2

(By Parts, Partial Fraction& three more special integrals)

Q: Integrate following functions with respect to x:

1. $\dfrac{x^2}{(x-1)(x+1)^2}$

2. $\cos^{-1}(4x^3-3x)$

3. $e^x\left[\dfrac{x-1}{(x+1)^3}\right]$

4. $\dfrac{\tan x+\tan^3 x}{1+\tan^3 x}$

5. $\left[\dfrac{\sqrt{1-\sin x}}{1+\cos x}\right]e^{-x/2}$

6. $\dfrac{x\tan^{-1}x}{(1+x^2)^{3/2}}$

7. $\dfrac{\log(x+2)}{(x+2)^2}$

8. $\dfrac{1}{\sin x(5-4\cos x)}$

9. $e^{\sqrt{x}}$

10. $\cos^3\sqrt{x}$

11. $e^{-3x}\cos 2x$

12. $e^x(\tan x-\log\cos x)$

13. $e^{2x}\dfrac{\sin 4x-2}{1-\cos 4x}$

14. $\dfrac{x^3-1}{x^3+x}$

15. $(x+1)\sqrt{3-x-x^2}$

16. $x^2\operatorname{cosec}^{-1}x$

17. $\left[\log(\log x)+\dfrac{1}{(\log x)^2}\right]$

18. $(3x+1)\sqrt{x^2+2x-3}$

SANSKRITI THE CIVIL SERVICES SCHOOL

Practice Questions

Integrate following functions w.r.t. x

1. $\frac{x}{x^4+x^2+1}$
2. $\frac{\sin x}{\sqrt{\cos^2 x-2\cos x-3}}$
3. $\frac{\sin^3 x}{\sqrt{\cos x}}$
4. $\frac{1}{1+\sqrt{x}}$
5. $\frac{1}{x\log x\log(\log x)}$
6. $\frac{1}{x(x^n+1)}$
7. $\frac{1}{\sqrt{x}+\sqrt[3]{x}}$
8. $\frac{1}{(3\sin x+\cos x)^2}$
9. $\frac{1}{sinx+\sqrt{3}\cos x}$
10. $\cos^7 x$
11. $\frac{1-x^2}{x(1-2x)}$
12. $\frac{\tan x}{\sqrt{\sin^4 x+\cos^4 x}}$
13. $\frac{\cos 2x}{\sin x}$
14. $\frac{\sin x+\cos x}{\sqrt{\sin 2x}}$
15. $\frac{1}{\sqrt{2x-x^2}}$
16. $\frac{x^2+5x+3}{x^2+3x+2}$
17. $\frac{2\sin 2x-\cos x}{6-\cos^2 x-4\sin x}$
18. $\frac{1}{4\sin^2 x+5\cos^2 x}$
19. $\frac{1}{3\sin^2 x+8\cos^2 x+1}$
20. $\frac{1}{2-3\cos 2x}$
21. $\frac{\cos x}{\cos 3x}$
22. $\frac{1}{\sec x+\cos ecx}$
23. $\frac{1}{(e^x+e^{-x})^2}$
24. $\frac{\sin 2x}{\sin^4 x+\cos^4 x}$
25. $\frac{1}{1+\cot x}$
26. $\sqrt{\cot\theta}$
27. $\frac{ax^3+bx}{x^4+c^2}$
28. $\sqrt{\frac{1+x}{x}}$
29. $(\sin^{-1} x)^3$
30. $\frac{x^2+x+1}{(x+1)^2(x+2)}$
31. $\frac{1}{\sin x(2+3\cos x)}$
32. $\frac{\cos x}{1+\cos x}$
33. $e^x\frac{(1-x)^2}{(1+x^2)^2}$
34. $\frac{1}{1+x+x^2+x^3}$
35. $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$
36. $\frac{1}{a+b\tan x}$

37. $\sqrt{1+x-2x^2}$

38. $\dfrac{\log(1-x)}{x^2}$

39. $e^x\left[\dfrac{1-\sin x}{1-\cos x}\right]$

40. $e^x\left[\dfrac{x-1}{(x+1)^3}\right]$

41. $(1-2x)\sqrt{4-3x-3x^2}$

42. $\left[\dfrac{\sqrt{1-\sin x}}{1+\cos x}\right]e^{-x/2}$

43. $\dfrac{x^3-1}{x^3+x}$

44. $(\log x)^2$

45. $\sin x \sin 2x \sin 3x$

46. $\dfrac{1}{\sin x(5-4\cos x)}$

47. $\dfrac{1}{(x-1)^2(x^2+2)}$

48. $\dfrac{\sin x}{(2+\cos x)(1-3\cos x)}$

विद्यया च विमुच्यते
SANSKRITI
THE CIVIL SERVICES SCHOOL

Assignment No. 8

Definite Integrals

Q1 - 8 are very short and short answer type questions.

1. Evaluate, $\int_0^{1.5}[x]\,dx$ (where [x] is greatest integer function)

2. If $\int_0^1\left(3x^2+2x+k\right)dx=0$, then find "$k$"

3. If $\int_0^a 3x^2dx=8$, then find the value of "a"

4. Evaluate $\int_{-1}^1|1-x|dx$

5. Evaluate $\int_0^1 e^{|x|}dx$

6. If $\int_0^k\frac{dx}{2+8x^2}=\frac{\pi}{16}$, find the value of k.

7. Evaluate $\int_0^{\pi/2}\frac{\sqrt{\cot x}}{1+\sqrt{\cot x}}dx$

8. If $\int_a^b x^3dx=0$ and $\int_a^b x^2\,dx=\frac{2}{3}$, find the value of a and b.

9. Evaluate the following definite integrals:

a) $\int_0^{\pi/4}2\tan^3 x\,dx$

b) $\int_0^{\pi/2}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

c) $\int_0^{\pi}\frac{dx}{6-\cos x}$

d) $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\cot^{3/2}x}$

e) $\int_0^{\pi}e^{\cos^2 x}\cos x\,dx$

f) $\int_0^{\pi/2}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

g) $\int_0^1\cot^{-1}\left(1-x+x^2\right)dx$

h) $\int_0^1\frac{\log(1+x)}{1+x^2}dx$

i) $\int_1^3\left|x^2-2x\right|dx$

j) $\int_0^{\pi}\frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$

k) $\int_{-5}^{0}(|x|+|x+2|+|x+5|)\,dx$

l) $\int_{-\pi}^{\pi}\frac{2x(1+\sin x)}{1+\cos^2 x}dx$

m) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{x+\frac{\pi}{4}}{2-\cos 2x}dx$

10. Evaluate following definite integrals as limit of a sum:

i. $\int_{0}^{1}\left(x^2-3x\right)dx$

iii. $\int_{-2}^{2}\left(3x^2-2x+4\right)dx$

ii. $\int_{1}^{3}e^{2x}dx$

iv. $\int_{0}^{4}\left(x+e^x\right)dx$

Learning Outcomes:

Students will be able to

- Accept integration as inverse process of differentiation
- Evaluate standard integrals by inspection.
- give geometrical interpretation of indefinite integral, discuss and use properties of indefinite integral
- use substitution method to find integral of a given function
- use trigonometric identities to integrate
- Integrate functions of special form

  $\int\frac{dx}{x^2\pm a^2}$, $\int\frac{dx}{a^2-x^2}$, $\int\frac{dx}{\sqrt{x^2\pm a^2}}$, $\int\frac{dx}{\sqrt{a^2-x^2}}$,

  $\int\frac{dx}{ax^2+bx+c}$, $\int\frac{dx}{\sqrt{ax^2+bx+c}}$, $\int\frac{px+q}{ax^2+bx+c}dx$,

  $\int\frac{px+q}{\sqrt{ax^2+bx+c}}dx$.

  . Use of partial fractions

  . Integration by parts

  . Integrate functions of the type

  $\int e^x\left[f(x)+f'(x)\right]dx$

  . Integrate functions of the type

  $\int\sqrt{a^2+x^2}dx$, $\int\sqrt{a^2-x^2}dx$,

  $\int\sqrt{x^2-a^2}dx$

- apply fundamental theorem of calculus to evaluate definite integrals
- Use substitution method along with fundamental theorem of calculus.
- Evaluate definite integral as a limit of sum
- Use properties of definite integrals to evaluate given integral

Assignment No. 9

Applications of Integrals-Area of the bounded regions

1. Find the area of the region bounded by the curve $y^2 = x-2, x=4, x=6$ and the x axis in the first quadrant using integration.

2. Sketch the region common to the circle $x^2 + y^2 = 16$ and the parabola $x^2 = 6y$. Also, find the area of the region using integration.

3. Find the area of the region bounded between the parabolas $y^2 = 4ax, x^2 = 4ay$, where a > 0.

4. Using Integration find the area of the triangle ABC whose vertices has coordinates given by $A(2,5), B(4,7), C(6,2)$

5. Compute the area bounded by the lines $x+2y=2,\ y-x=1\ \&\ 2x+y=7$

6. Find the area of the region bounded by the curve $y^2 = 2y - x$ and the y axis.

7. Using integration find the area of the region $\{(x,y): |x| \le y \le \sqrt{4-x^2}\ \}$

8. Find the area bounded by the triangle whose vertices are $(0,0),(2,4),(4,-2)$.

9. Sketch the graph of $f(x) = \begin{cases} |x-2|+2, x \le 2 \\ x^2-2, x > 2 \end{cases}$.Evaluate $\int_0^4 f(x)dx$, what does the value of this integral represent on the graph?

10. Find the area bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$

11. Find the area of the region $\{(x,y): x^2 + y^2 \le 1 \le x + y\}$

12. Find the area of the region enclosed between the circles $x^2 + y^2 = 4, (x-2)^2 + y^2 = 1$.

13. Using integration, find the area of the triangle formed by the tangent and the normal to the curve $y^2 = -4x$ at the point (-1, 2) and the x-axis.

14. Using integration find the area of the region bounded by the parabola $y^2 = 2x$ and the line $x - y = 4$

15. Using integration find the area of the region $\{(x,y): |x-1| \le y \le \sqrt{5-x^2}\}$

Learning Outcomes:

Students will be able to

- draw simple curves and identify the region whose area is to be evaluated
- use integration to find area under simple curves
- sketch a rough graph, identify and evaluate area enclosed between two curves and to find area of the region bounded by a curve and a line

विद्यया च विमुच्यते
SANSKRITI
THE CIVIL SERVICES SCHOOL

Differential Equations

Find the general solution:	Answer:
Q1. $\sqrt{a+x}\,\frac{dy}{dx}=-xy$	$\log y+\frac{2}{3}(a+x)^{3/2}-2a\sqrt{a+x}=c$
Q2. $\left[x\sqrt{x^2+y^2}-y^2\right]dx+xydy=0$	$\sqrt{x^2+y^2}=x\log\left\lvert\frac{c}{x}\right\rvert$
Q3. $\left(y-x\frac{dy}{dx}\right)x=y$	$\log\left\lvert\frac{x}{y}\right\rvert+\frac{1}{x}=c$
Q4. $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$	$e^{x/y}+\frac{x}{y}=\frac{c}{y}$
Q5. $y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$	$(x+a)(1-ay)=cy$
Q6. $\frac{dy}{dx}=e^{x+y}+x^2e^{x^3+y}$	$-e^{-y}=e^x+\frac{1}{3}e^{x^3}+C$
Q7. $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$	$\sqrt{3}(x+y+1)=c(1-x-y-2xy)$
Q8. $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$	$\sqrt{1+x^c}+\sqrt{1+y^2}=\log\frac{1+\sqrt{1+x^2}}{x}+c$
Q9. $x\frac{dy}{dx}=y(\log y-\log x+1)$	$\log\frac{y}{x}=cx$
Q10. $\left(x\cos\frac{y}{x}+y\sin\frac{y}{x}\right)y-\left(y\sin\frac{y}{x}-x\cos\frac{y}{x}\right)x\frac{dy}{dx}=0$	$\sec\frac{y}{x}=cxy$
Q11. $\frac{dy}{dx}=\frac{x+y+1}{x+1}$	$\frac{y}{x+1}=\log(x+1)+c$
Q12. $x\frac{dy}{dx}-y=(x-1)e^x$	$y=e^x+cx$
Q13. $\left(1+y^2\right)+\left(x-e^{-\tan^{-1}y}\right)\frac{dy}{dx}=0$	$xe^{tan^{-1}y}=tan^{-1}y+C$

Q14. $x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)=1$	$y=\frac{\sin x}{x}+\frac{c\cos x}{x}$
Q15. $ydx+(x-y^2)dy=0$	$x=\frac{y^2}{3}+\frac{c}{y}$
Q16. $\frac{dy}{dx}=1-x+y-xy$	$\log\lvert 1+y\rvert=x-\frac{x^2}{2}+c$
Q17. $(x+y+1)\frac{dy}{dx}=1$	$(x+y+1)-\log\lvert x+y+2\rvert=x+c$
Q18. $\frac{dy}{dx}+1=e^{x-y}$	$\frac{-1}{2}\log\lvert 2e^{y-x}-1\rvert=x+c$
Q19. $(x\log x)\frac{dy}{dx}+y=\frac{2}{x}\log x$	$y\log x=\frac{-2}{x}(\log x+1)+c$
Q20. $\log(\frac{dy}{dx})=ax+by$	$-\frac{1}{b}e^{-by}=\frac{1}{a}e^{ax}+c$
Q21. $(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}})\frac{dx}{dy}=1$	$ye^{2\sqrt{x}}=2\sqrt{x}+c$
Q22. $xdy-ydx=\sqrt{x^2+y^2}dx$	$y+\sqrt{x^2+y^2}=cx^2$
Q23. $\frac{dy}{dx}+1=e^{x+y}$	$(x+C)e^{x+y}+1=0$
Q24. $y+\frac{d}{dx}(xy)=x(\sin x+\log x)$	$y=-\cos x+\frac{2\sin x}{x}+\frac{2\cos x}{x^2}+\frac{x}{3}\log x-\frac{x}{9}+cx^{-2}$
Q25. Form the differential equation having $y=(\sin^{-1}x)^2+A\cos^{-1}x+B$, where A and B are arbitrary constants, as its general solution	$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0$
Q26. Form the differential equation of system of concentric circles with centre (1,2)	$(y-2)\frac{dy}{dx}+(x-1)=0$

THE CIVIL SERVICES SCHOOL

	Find the particular solution:	Answer:
Q27.	$\left(1+y^2\right)(1+\log x)dx + xdy = 0,\ y(1)=1$	$\log\|x\| + \frac{(\log\|x\|)^2}{2} + \tan^{-1} y = \frac{\pi}{4}$
Q28.	$\left(1+\sin^2 x\right)dy + (1+y^2)\cos x dx = 0,\ y(\frac{\pi}{2}) = 0$	$\tan^{-1} y + \tan^{-1}(\sin x) = \frac{\pi}{4}$
Q29.	$x\frac{dy}{dx}\sin(\frac{y}{x}) + x - y\sin(\frac{y}{x}) = 0,\ \ y(1) = \frac{\pi}{2}$	$\cos(\frac{y}{x}) = \log\|x\|$
Q30.	$\frac{dy}{dx} = \frac{2x+y-1}{4x+2y+5},\ y(0)=0$	$\frac{2}{5}(2x+y) + \frac{7}{25}\log\|10x+5y+9\| = x + \frac{7}{25}\log 9$
Q31.	$\left(3x^2+y\right)\frac{dx}{dy} = x > 0,\ y(1)=1$	$y = 3x^2 - 2x$
Q32.	$x\frac{dy}{dx} + \frac{y}{\log x} = 1,\ y(1) = 1$	$y = \frac{1}{2}\log\|x\|$
Q33.	$dy = \cos x\left(2 - y\cos ecx\right)dx\ ;\ y\left(\frac{\pi}{2}\right) = 2$	$y\sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}$
Q34.	$\left(x+2y^2\right)\frac{dy}{dx} = y,\ y(2) = 1$	$x = 2y^2$
Q35.	$\sqrt{1-y^2}\,dx = \left(\sin^{-1} y - x\right)dx,\ y(0) = 0$	$x + 1 - \sin^{-1} y = e^{-\sin^{-1} y}$

Assignment No. 10

Differential Equations

Q1-4 and each part of Q5&6 are very short and short answer type question.

1. Find the differential equation corresponding to $y = Ae^{x} + Be^{-x}$

2. Find a solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$.

3. Show that the differential equation $x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$ is homogenous.

4. Determine the order and degree (if defined) of each of the following differential equations:

   i. $\left(\frac{d^2x}{dt^2}\right)^4 - 7t\left(\frac{dx}{dt}\right)^3 = \log t$

   ii. $\left(\frac{d^2y}{dx^2}\right)^2 + \sin\left(\frac{dy}{dx}\right) = 0$

   iii. $1 + \left(\frac{dy}{dx}\right)^2 = 2x - \frac{dy}{dx}$

   iv. $\left[1 + \left(\frac{dy}{dx}\right)^2\right] = 5\left(\frac{d^2y}{dx^2}\right)^3$

5. Write the integrating factor of the following differential equations:

   i. $(1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x$

   ii. $x\frac{dy}{dx} - y = \log x$

   iii. $\frac{dx}{dy} - \frac{2x}{y} = 3y^3 - 5y + 1$

   iv. $\frac{dy}{dx} + 2y = xe^{4x}$

6. Solve the following differential equations:

i. $(x-1)\frac{dy}{dx} = 2xy$, given that $x = 2, y = 1$

ii. $(1+x)\,ydx + (1+y)\,xdy = 0$

iii. $\frac{dy}{dx} + 2y = e^{-2x}\sin x$, given $x = 0, y = 0$

iv. $ydx - (x + 2y^2)dy = 0$, given that x=2 when y = 1.

v. $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$

vi. $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$

SANSKRITI THE CIVIL SERVICES SCHOOL

vii. $(x+y+1)\frac{dy}{dx}=1$

viii. $\left(1+e^{2x}\right)dy+\left(1+y^2\right)e^x dx=0,$ given that $x=0, y=1$

ix. $\frac{dy}{dx}=1-x+y-xy$

Learning Outcomes:

Students will be able to

- Identify a differential equation.
- Know the order and degree of the differential equation
- How to form the differential equation when the general solution of the equation is given
- To verify if a given function (explicit or implicit) is a solution of the corresponding differential equation.
- Identify the form of differential equation.
- Solve a differential equation of the variable separable form.
- Show a differential equation is a homogenous one and to solve it.
- Find the integrating factor of the differential equation.
- Solve the linear differential equation after finding the integrating factor.
- Appreciate the differential equation in real life situations.

SANSKRITI THE CIVIL SERVICES SCHOOL

Vectors

1. If $\left|\vec{a}\right| = 3$ and $-2 \le k \le 1$, then what can you say about $\left|k\vec{a}\right|$?
2. For what values of 'a' the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.
3. Find a vector of magnitude 11 units in the direction opposite to $\overrightarrow{PQ}$ where P and Q are the points $(1,3,2)$ and $(-1,0,8)$ respectively.
4. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{2}$ with y-axis and an acute angle θ with z-axis.
5. A vector $\vec{r}$ has magnitude 14 units and direction ratios $2, 3, -6$. Find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an acute angle with x-axis.
6. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find unit vectors parallel to the vector $\vec{a} + \vec{b}$.
7. Prove that the points with position vectors $\hat{i} - \hat{j}, 4\hat{i} - 3\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + 5\hat{k}$ are the vertices of a right-angled triangle.
8. Find the position vectors of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in the ratio 1:2. Also, show that P is the mid-point of the line segment RQ.
9. If the vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent two side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC, then find the length of median through A.
10. If the points $A(-1,-1,2), B(2,m,5)$ and $C(3,11,6)$ are collinear, then find the value of m by vector method.
11. If $\vec{a}$ is a unit vector and $(2\vec{x} - 3\vec{a}).(2\vec{x} + 3\vec{a}) = 91$, then find the value of $\left|\vec{x}\right|$.
12. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $(\vec{a} + \vec{b})$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.
13. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, then show that the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other.
14. The vectors $\vec{a} = 3\hat{i} + x\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular. If $\left|\vec{a}\right| = \left|\vec{b}\right|$, then find the value of y.
15. Find λ when the scalar projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units.
16. If A, B, C, D are points with position vectors $\hat{i} + \hat{j} - \hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$, $2\hat{i} - 3\hat{k}$, $3\hat{i} - 2\hat{j} + \hat{k}$ respectively, find the projection of $\overrightarrow{AB}$ along $\overrightarrow{CD}$.

SANSKRITI THE CIVIL SERVICES SCHOOL

17. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $(\vec{b} + \vec{c})$.
18. Three vertices of a triangle are $A(0,-1,-2)$, $B(3,1,4)$ and $C(5,7,1)$. Show that it is a right angled triangle. Also, find the other two angles.
19. If $\vec{a}$ and $\vec{b}$ are two non-zero, non-collinear vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, then prove that $(2\vec{a} + \vec{b})$ is perpendicular to $\vec{b}$.
20. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $(\vec{a} + \vec{b} + \vec{c}) = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.
21. Find vectors of magnitude $10\sqrt{3}$ units that are perpendicular to the plane of vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-\hat{i} + 3\hat{j} + 4\hat{k}$.
22. Find a unit vector perpendicular to the plane of triangle ABC where the coordinates of its vertices are $A(3,-1,2)$, $B(1,-1,-3)$ and $C(4,-3,1)$.
23. Find the angle between the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ if $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$ and hence find a vector perpendicular to both $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$.
24. Find the value of $\vec{a}.\vec{b}$, if $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $|\vec{a} \times \vec{b}| = 16$.
25. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then show that $(\vec{a} - \vec{d})$ is parallel to $(\vec{b} - \vec{c})$ where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.
26. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.
27. Prove the following:
    (i) $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$
    (ii) $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}] = 0$
28. Show that the four points A,B,C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively are coplanar.
29. Prove that: $\vec{a}.(\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}]$.
30. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then prove that:
    (i) $\vec{a} = \pm 2(\vec{b} \times \vec{c})$
    (ii) $[\vec{a} + \vec{b}\quad \vec{b} + \vec{c}\quad \vec{c} + \vec{a}] = \pm 1$

Assignment No.11 (a)

VECTORS

Very short and short answer type questions

1. Write down a unit vector in XY plane making an angle of 60^0 with the positive direction of x-axis.
2. Find the distance of the point (a,b,c) from the z- axis.
3. Give an example of two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = |\vec{b}|$ but $\vec{a} \neq \vec{b}$.
4. If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 6\hat{i} + \lambda\hat{j} + 3\hat{k}$, such that they are collinear vectors, then find λ
5. Write the direction cosines of the vector $3\hat{i} - 6\hat{j} + 2\hat{k}$.
6. If the magnitude of the position vector of the point (3, -2, p) is 7 units, find all possible values of p.
7. A vector is inclined at $\frac{\pi}{4}$, $\frac{\pi}{3}$ with the "x" and "y" axes respectively. Find the angle it makes with the "z" axis.
8. If $\vec{a} = p\hat{i} + 3\hat{j}$ and $\vec{b} = 4\hat{i} + p\hat{j}$, find the values of p so that $\vec{a}$ and $\vec{b}$ may be parallel.
9. Write the position vector of the point dividing the line segments joining the points with position vectors $\vec{a}$ and $\vec{b}$ in the ratio 1 : 4 externally, where $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$
10. Find the angle at which the following vectors are inclined to each of the coordinate axes: (i) $\hat{i} + \hat{j} - \hat{k}$ (ii) $-\hat{i} - \hat{j}$
11. Show that $\cos\alpha\cos\beta\,\hat{i} + \cos\alpha\sin\beta\,\hat{j} + \sin\alpha\,\hat{k}$ is a unit vector.
12. Find a unit vector perpendicular to the vectors $\hat{i} - 3\hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + \hat{k}$.
13. Find a vector of magnitude 3 units, which is orthogonal to the vectors $3\hat{i} + \hat{j} - 4\hat{k}$ and $6\hat{i} + 5\hat{j} - 2\hat{k}$.
14. If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ & $\vec{b} = -\hat{i} + 3\hat{j} - 2\hat{k}$, then find $|\vec{a} - 2\vec{b}|$
15. Find the scalar and vector projection of $2\hat{i} - \hat{j} + \hat{k}$ on $\hat{i} - 2\hat{j} + \hat{k}$.
16. Show that the three points A (3, -5 , 1) B (-1 , 0 , 8) and C (7 , -10 ,-6) are collinear.

SANSKRITI THE CIVIL SERVICES SCHOOL

17. For what value of 'p', the vectors $\vec{a} = 3\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = p\hat{i} + 3\hat{j} + 3\hat{k}$ are perpendicular to each other?

18. If $\vec{a} = x\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + y\hat{k}$, find the value of x and y so that $\vec{a}$ and $\vec{b}$ may be collinear.

19. Find the position vector of the midpoint of the line segment joining the points $A(5\hat{i} + 3\hat{j})$ and $B(3\hat{i} - \hat{j})$

20. If $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, find $\vec{a}.\vec{b}$.

21. A line makes angles of 45° and 45° with x and y axis respectively. Find the angle it makes with z-axis.

22. Evaluate: $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{k} \times \hat{i}) + \hat{k}.(\hat{j} \times \hat{i})$

SANSKRITI
THE CIVIL SERVICES SCHOOL

Assignment No.11(b)

VECTORS

1. The vectors $\vec{a}$ and $\vec{b}$ are non - zero non- collinear and $\vec{c} = (p+4q)\vec{a} + (2p+q+1)\vec{b}$, and $\vec{d} = (-2p+q+2)\vec{a} + (2p-3q-1)\vec{b}$ then find the values of p and q so that $3\vec{c} = 2\vec{d}$

2. If $\vec{a}$ and $\vec{b}$ are unit vectors , then what is the angle between $\vec{a}$ and $\vec{b}$ so that $\vec{a} - \sqrt{2}\,\vec{b}$ is a unit vector?

3. If $|\vec{a}|=3$, $|\vec{b}| = 5$ and $\vec{a}.\vec{b} = -8$, find $|\vec{a} + \vec{b}|$.

4. The adjacent sides of a parallelogram are represented by the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} + 2\hat{k}$.Find unit vectors parallel to the diagonals of the parallelogram.

5. Compute area of a parallelogram whose diagonals are the vectors $2\hat{i} - 3\hat{j} + 6\hat{k}$ and $2\hat{i} - 2\hat{j} - \hat{k}$.

6. If A, B, C have position vectors (2, 0 ,0), (0, 1 , 0) , (0, 0 , 2), show using vectors that triangle ABC is isosceles.

7. Determine λ and μ if $\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) \times \left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right) = \vec{0}$

8. Dot product of a vector with vectors $\hat{i} + \hat{j} + \hat{k}, \hat{i} + 2\hat{j} + 3\hat{k}$ and $\hat{i} + 3\hat{j} + 4\hat{k}$ is respectively 7, 16 and 22. Find the vector.

9. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$,prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

10. Find the volume of the parallelopiped whose sides are given by $-3\hat{i} + 7\hat{j} + 5\hat{k}$, $-5\hat{i} + 7\hat{j} - 3\hat{k}$ & $7\hat{i} - 5\hat{j} - 3\hat{k}$

11. Show that the vectors $-2\hat{i} - 2\hat{j} + 4\hat{k}$, $-2\hat{i} + 4\hat{j} - 2\hat{k}$ and $4\hat{i} - 2\hat{j} - 2\hat{k}$ are coplanar.

12. Find "λ", for which the four points with position vectors $-\hat{j} - \hat{k}$, $4\hat{i} + 5\hat{j} + \lambda\hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.

13. Prove that $\left(\vec{a} - \vec{b}\right) \cdot \left\{\left(\vec{b} - \vec{c}\right) \times \left(\vec{c} - \vec{a}\right)\right\} = 0$

14. Evaluate: $\left[2\hat{i}\ \hat{j}\ \hat{k}\right] + \left[\hat{i}\ \hat{k}\ \hat{j}\right] + \left[\hat{k}\ \hat{j}\ 2\hat{i}\right]$

15. For any three vectors $\vec{a}, \vec{b}, \vec{c}$, prove that $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right]=2\left[\vec{a}, \vec{b}, \vec{c}\right]$

16. Determine "α" such that a vector $\vec{r}$, is at right angles to each of the three vectors $\vec{a}=\alpha\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}$ and $\vec{c}=-2\hat{i}+\hat{j}+3\hat{k}$

17. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}$. Prove that $\vec{a}, \vec{b}, \vec{c}$ are mutually at right angles and $\left|\vec{b}\right|=1, \left|\vec{a}\right|=\left|\vec{c}\right|$.

18. *If* $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}, \vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, Prove that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, provided $\vec{a}\neq\vec{d}$, and $\vec{b}\neq\vec{c}$.

19. Find two vectors of unit length which make angles of $45°$ with the position vector of(1 , 0, 0) and are at right angles to the position vector of (0, 0 , 1).

20. Find vector $\vec{c}$ such that $\vec{c}.\hat{i}=\vec{c}.\hat{j}=\vec{c}.\hat{k}$ and $\left|\vec{c}\right|=100$.

Learning Outcomes:

Students will be able to

- Define a vector
- Distinguish between a vector and a scalar.
- Various types of vectors
- Find the magnitude of a vector and to find a vector of given magnitude in the given direction
- Do the operations on a vector
- Use the section formula and the distance formula in a vector
- To find the vector joining two points
- Find the scalar product of two vectors
- Find the projection of one vector on the other
- Vector product of two vectors
- Find the area of a parallelogram
- Find the angle between two vectors
- scalar triple product and coplanarity of three vectors

Three-Dimensional Geometry

1. A straight line makes angles $60°$ and $45°$ with the positive directions of X-axis and Y-Axis respectively. What angle does it make with the Z-Axis?
2. Find the direction cosines of the line passing through the two points $(-2,4,-5)$ and $(1,2,3)$.
3. For what values of p and q will the line joining the points $A(3,2,5)$ and $B(p,5,0)$ be parallel to the line joining points $C(1,3,q)$ and $D(6,4,-1)$.
4. Find the coordinates of the foot of perpendicular drawn from the point $A(-1,8,4)$ to the line joining the points $B(0,-1,3)$ and $C(2,-3,1)$. Hence find the image of the point A in the line BC.
5. The equations of a line are $5x-3=15y+7=3-10z$. Write the direction cosines of the line.
6. The Cartesian equations of a line are $2x-3=3y+1=5-6z$. Find the direction ratios of the line and write down the vector equation of the line through $(7,-5,0)$ which is parallel to the given line.
7. The points $A(1,2,3), B(-1,-2,-1)$ and $C(2,3,2)$ are three vertices of a parallelogram ABCD. Find vector and Cartesian equations of the sides AB and BC. Also find the coordinates of D.
8. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point $P(1,3,3)$.
9. Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}, z+1=0$ and $\frac{x-4}{2}=\frac{z+1}{3}, y=0$ intersect. Also, find their point of intersection.
10. Find the angle between the lines $2x=3y=-z$ and $6x=-y=-4z$.
11. Find the value of p, so that the lines $l_1 : \frac{1-x}{3}=\frac{7y-14}{p}=\frac{z-3}{2}$ and $l_2 : \frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other. Also, find the equations of a line passing through the point $(3,2,-4)$ and parallel to the line l_1 .
12. Find the image of the point $P(1,6,3)$ in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.
13. A line passing through the point A with position vector $\vec{a}=4\hat{i}+2\hat{j}+2\hat{k}$ is parallel to the vector $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$. Find the length of perpendicular drawn on this line from a point P with position vector $\vec{r_1}=\hat{i}+2\hat{j}+3\hat{k}$.

14. Find the vector and cartesian equations of a line through the point $(1,-1,1)$ and perpendicular to the lines joining the points $(4,3,2),(1,\text{-}1,0)$ and $(1,2,\text{-}1),(2,1,1)$.
15. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angles of $\frac{\pi}{3}$.
16. Find the shortest distance between the lines $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{3-x}{-1}=\frac{y-5}{-2}=\frac{z-7}{1}$.
17. Find the shortest distance between the following lines: $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and $\vec{r}=2\hat{i}+4\hat{j}+5\hat{k}+\mu(4\hat{i}+6\hat{j}+8\hat{k})$
18. Find the equations of the line which intersects the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and passes through the point $(1,1,1)$.
19. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i}-3\hat{j}+6\hat{k}$.
20. Find the vector and cartesian equations of the plane which bisects the line joining the points $(3,-2,1)$ and $(1,4,\text{-}3)$ at right angles.
21. Show that the line $\vec{r}=4\hat{i}-7\hat{k}+\lambda(4\hat{i}-2\hat{j}+3\hat{k})$ is parallel to the plane $\vec{r}.(5\hat{i}+4\hat{j}-4\hat{k})=7$.
22. Show that the line $\vec{r}=2\hat{i}+3\hat{j}+\lambda(7\hat{i}-5\hat{k})$ lies in the plane $\vec{r}.(5\hat{i}-3\hat{j}+7\hat{k})=1$.
23. Find the distance of the point $A(-1,-5,-10)$ from the point of intersection of the line $\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$.
24. Find the coordinates of the point where the line $\vec{r}=(-\hat{i}-2\hat{j}-3\hat{k})+\lambda(3\hat{i}+4\hat{j}+3\hat{k})$ meets the plane which is perpendicular to the vector $\vec{n}=\hat{i}+\hat{j}+3\hat{k}$ at a distance of $\frac{4}{\sqrt{11}}$ from origin.
25. Find the distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured along a line parallel to the line $\frac{x-1}{2}=\frac{y-3}{3}=\frac{z+2}{-6}$.
26. Find the distance of the point $A(-2,3,-4)$ from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$.

SANSKRITI THE CIVIL SERVICES SCHOOL

27. Find the length and the foot of the perpendicular from the point $\left(1,\frac{3}{2},2\right)$ to the plane $2x-2y+4z+5=0$.
28. Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i}+3\hat{j}+4\hat{k}$ to the plane $\vec{r}.(\ 2\hat{i}+\hat{j}+3\hat{k}\ )-26=0$. Also find image of P in the plane.
29. Find the equation of the plane passing through the points $(1,2,3),(0,-1,0)$ and parallel to the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-3}$.
30. Find the equation of the plane which is parallel to X-Axis and has intercepts 5 and 7 on y-axis and z-axis respectively.
31. Find the equation of the plane passing through the point $2\hat{i}-\hat{k}$ and parallel to the lines $\frac{x}{-3}=\frac{y-2}{4}=z+1$ and $x-4=\frac{1-y}{2}=2z$.
32. Show that the lines $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ are coplanar. Also find the equation of the plane containing these lines.
33. Find the equation of the plane containing two parallel lines $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{3}$ and $\frac{x}{4}=\frac{y-2}{-2}=\frac{z+1}{6}$. Also find if the plane thus obtained contains the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{5}$ or not.
34. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then find the value of k and hence find the equation of the plane containing these lines.
35. Find the equation of the plane passing through the points $(3,4,2),(2,-2,-1)$ and $(7,0,6)$.
36. Find the coordinates of the point P where the line through $A(3,-4,-5)$ and $B(2,-3,1)$ crosses the plane passing through three points $L(2,2,1), M(3,0,1)$ and $N(4,-1,0)$. Also find the ratio in which P divides the line segment AB.
37. Find the equation of the plane which cuts off intercepts $3,-4$ and 6 from the axes. Reduce it to normal form and hence find the length of perpendicular from origin to the plane.
38. If a plane meets the coordinate axes in points A,B,C and the centroid of the triangle ABC is (α,β,γ), find the equation of the plane.
39. Find the equations of the two planes passing through the points $(0,4,-3)$ and $(6,-4,3)$, if the sum of their intercepts on the three axes is zero.

THE CIVIL SERVICES SCHOOL

40. Find the equation of the plane passing through the line of intersection of the planes $x+2y+3z-5=0$ and $3x-2y-z+1=0$ and cutting off equal intercepts on x-axis and z-axis.
41. Find the angle between the planes whose vector equations are $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=5$ and $\vec{r}.(3\hat{i}-3\hat{j}+5\hat{k})=3$.
42. Find the angle between the line $\frac{x-2}{3}=\frac{y-3}{5}=\frac{z-4}{4}$ and the plane $2x-2y+z-5=0$.
43. Find the coordinates of the point where the line through the points $A(3,4,1)$ and $B(5,1,6)$ crosses the XZ plane. Also find the angle which this line makes with the XZ plane.
44. Find the direction ratios of a normal to the plane, which passes through the points $(1,0,0)$ and $(0,1,0)$ and makes angle $\frac{\pi}{4}$ with the plane $x+y=3$. Also find the equation of the plane.
45. Find the vector equation of the line passing through $(1,2,3)$ and parallel to the planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}.(3\hat{i}+\hat{j}+\hat{k})=6$.
46. Find the equation of the plane through the point $(4,-3,2)$ and perpendicular to the line of intersection of the planes $x-y+2z-3=0$ and $2x-y-3z=0$. Find the point of intersection of the line $\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda(\hat{i}+3\hat{j}-9\hat{k})$ and the plane obtained above.
47. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})+5=0$ which is perpendicular to the plane $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.
48. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$ and $5x-3y+4z-9=0$ and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{5-z}{-5}$.
49. Find the distance of the point $(2,5,-3)$ from the plane $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})=4$.
50. Find the Cartesian as well as vector equations of the planes through the intersection of the planes $\vec{r}.(\hat{i}+3\hat{j})-6=0$ and $\vec{r}.(3\hat{i}-\hat{j}+4\hat{k})=0$ which are at a unit distance from the origin.
51. Find the distance between the planes $\vec{r}.(2\hat{i}-\hat{j}+2\hat{k})=6$ and $\vec{r}.(6\hat{i}-3\hat{j}-6\hat{k})=27$.
52. Find the equation of the plane mid parallel to the planes $2x-3y+6z+21=0$ and $2x-3y+6z-14=0$.

Assignment No. 12

Three-Dimensional Coordinate Geometry

Q1-10 are very short and short answer type questions.

1. Find the perpendicular distance of the plane $\vec{r}.(5\hat{i}-3\hat{j}+4\hat{k})+9=0$ from origin.
2. Find vector equation of the plane which is at a distance of 3 units from origin and has $\hat{j}$ as the unit normal.
3. Write the equation of the plane passing through the point $(2,-1,1)$ and parallel to the plane $3x+2y-z=7$
4. If the lines $\frac{x-1}{-3}=\frac{y-2}{2p}=\frac{z-3}{2}$ and $\frac{x-1}{3p}=\frac{y-1}{1}=\frac{z-6}{-5}$ are perpendicular to each other, then find the value of p.
5. Find the equation of the line passing through the point $2\hat{i}-3\hat{j}+4\hat{k}$ parallel to the line $\vec{r}=\hat{i}-3\hat{j}-5\hat{k}+\lambda(2\hat{i}+5\hat{k})$.
6. Write position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ & $\vec{b}$ externally in the ratio 1: 4, where $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}+\hat{k}$
7. The cartesian equations of a line are $3x+1=6y-2=1-z$. Find the fixed point through which it passes and its direction ratios. Also find the vector equation.
8. Find the angle between $\frac{x-1}{2}=\frac{2-y}{-1}=\frac{-z-3}{2}$ and $x+y+4=0$
9. Find the intercepts cut off by the plane $3x+2y+z=7$.
10. Write the equation of the plane parallel to XOY plane passing through the point $(1,-2,5)$
11. Find the foot of the perpendicular from P (1, 2, 3) on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Also obtain the equation of the plane containing the line and the point (1, 2 , 3).
12. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)=3$ and $\vec{r}.\left(5\hat{i}-3\hat{j}+4\hat{k}\right)+9=0$ and parallel to the line $\vec{r}=\hat{i}+3\hat{j}+5\hat{k}+\lambda(2\hat{i}+4\hat{j}+5\hat{k})$.

13. Find the equation of the plane through the points (1 , 0 , -1) , (3, 2, 2) and parallel to the line $\frac{x-1}{1}=\frac{y-1}{-2}=\frac{z-2}{3}$.

14. Find the shortest distance between the lines:

a) $\vec{r}=(-4\hat{i}+4\hat{j}+\hat{k})+\lambda(\hat{i}+\hat{j}-\hat{k})$
$\vec{r}=(-3\hat{i}-8\hat{j}-3\hat{k})+\mu(2\hat{i}+3\hat{j}+3\hat{k})$

b) $\vec{r}=(3+\lambda)\hat{i}+(5-2\lambda)\hat{j}+(7+\lambda)\hat{k})$
$\vec{r}=(7\mu-1)\hat{i}+(-1-6\mu)\hat{j}+(\mu-1)\hat{k}$

15. Find the equation of the plane through the points (2,1, -1) , (-1, 3, 4) and perpendicular to the plane $x-2y+4z=10$

16. Show that the four points (0, -1 , -1) , (4, 5 , 1) , (3, 9, 4) and (-4, 4, 4) are coplanar. Also find the equation of the plane containing them.

17. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i}+3\hat{j})+6=0$ and $\vec{r}.(3\hat{i}-\hat{j}-4\hat{k})=0$, which is at a unit distance from the origin.

18. Find the equation of the plane passing through the line of intersection of the planes $4x-y+z=10$ and $x+y-z=4$ and parallel to the line with direction ratios proportional to 2, 1, 1. Find also the perpendicular distance of (1, 1, 1) from this plane.

19. Show that the lines $\vec{r}=2\hat{j}-3\hat{k}+\lambda(\hat{i}+2\hat{j}+3\hat{k})$ and $\vec{r}=2\hat{i}+6\hat{j}+3\hat{k}+\mu(2\hat{i}+3\hat{j}+4\hat{k})$ are coplanar. Also find the equation of the plane containing them.

20. Prove that the image of the point (3,-2,1) in the plane $3x-y+4z=2$ lies on the plane $x+y+z+4=0$.

21. Find the distance of the point (3,4,5) from the plane $x+y+z=2$ measured parallel to the line $2x=y=z$

22. Show that the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$ is parallel to the plane $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$ .Also, find the distance between them.

23. Find the coordinates of the point where the line through (5, 1 ,6) and (3, 4, 1) crosses the ZX-plane .

24. Find the equations of the planes parallel to the plane $x-2y+2z-3=0$ and which are at a unit distance from the point (1,1,1).

25. Find the equation of the line which intersects the lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and passes through the point (1, 1, 1).

26. Find the equation of the plane through the point (4, -3, 2) and perpendicular to the line of intersection of the planes $x-y+2z-3=0$ and $2x-y-3z=0$. Find the point of intersection of the line $\vec{r}=\hat{i}+2\hat{j}-\hat{k}+\lambda(\hat{i}+3\hat{j}-9\hat{k})$ and the plane obtained above.

Learning Outcomes:

Students will be able to

- find the direction cosine and direction ratio of a vector and a line
- find the equation of a line passing through a point and parallel to a vector (vector and Cartesian)
- find the equation of a line passing through two points (vector and cartesian)
- find the angle between two lines
- recognize two skew lines
- find shortest distance between two skew lines and two parallel lines.
- find the foot of the perpendicular of a point on a line
- Find the Image of a point with a line as a plane mirror
- Know the standard equation of a plane
- Know the equation of a plane in normal and intercept form (both Cartesian and vector)
- To find the equation of a plane through three non collinear points (vector and Cartesian) and to find the equation of a plane perpendicular to a given vector through a given point. (vector and Cartesian form)
- To find the equation of a plane through intersection of two planes and with the given condition
- To find Coplanarity of two lines
- Find the Angle between two planes
- Find the Distance of a point from a plane and to find the foot of the perpendicular of a point in a plane. Also, to find the image of a point with a plane as mirror.
- To find the angle between the plane and a line

Probability

1. A die is thrown three times. If the first throw is a four, find the chances of getting 12 as the sum.
2. Two numbers are selected at random from numbers 1 to 11. If the sum is even, then find the probability that both numbers are odd.
3. A coin is tossed, then a die is thrown. Find the probability of obtaining a "6" given that head came up.
4. If P(A) = 3/8, P(B)=1/2, $P(A \cap B)$ = ¼, find $P(\overline{A}/\overline{B})$
5. If A and B are two events associated with same random experiment such that $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$ then find $P(A/B)$, $P(A \cup B)$ and $P(\overline{B}/\overline{A})$.
6. If $P(E) = \frac{7}{13}$, $P(F) = \frac{9}{13}$, $P(E \cap F) = \frac{4}{13}$, evaluate $P(E/F)$, $P(\overline{E}/F)$, $P(E/\overline{F})$ & $P(\overline{E}/\overline{F})$
7. A coin is tossed once. If it shows head, it is tossed again and if it shows tail, then a die is tossed. Let E: the first throw shows a tail and F: the die shows a number greater than 4. Find $P(F/E)$.
8. Two balls are drawn one after another (without replacement) from a bag containing 2 white, 3 red and 5 blue balls. What is the probability that at least one ball is red?
9. A bag contains 50 tickets numbered 1, 2, 3, … ,50 of which five are drawn at random and arranged in ascending order of the numbers appearing on the tickets $x_1 < x_2 < x_3 < x_4 < x_5$. Find the probability that $x_3 = 30$.
10. If the probability for A to fail in an examination is 0.2 and that for B is 0.3, find the probability that neither fails.
11. A problem in mathematics is given to three students A, B and C. Their chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{6}$ respectively. Find the probability (i) that the problem will be solved (ii) that exactly one of them solves the problem (iii) problem is not solved.
12. If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$, then find P(A) and P(B).

SANSKRITI THE CIVIL SERVICES SCHOOL

13. A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6. Find the probability that B is selected.
14. A , B and C in order throw a die in succession till one gets a "six" and wins the game. Find their respective probabilities of winning.
15. A and B throw a pair of dice alternately. A wins if he throws a 6 before B throws a 7 and B wins if he throws a 7 before A throws 6. If A begins, then show that his chances of winning are 30/61.
16. Two thirds of the students of a class are boys and the rest are girls. It is known that the probability of a girl getting A1 grade in Board Exam is 0.4 and a boy getting A1 grade is 0.35. Find the probability that a student chosen at random will get A1 grade in Exam.
17. There are three urns containing 3 white & 2 black balls, 2 white & 3 black balls and 1 black & 4 white balls. There is equal probability of each urn being chosen. One ball is drawn from an urn chosen at random. What is the probability that a white ball is drawn.
18. One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.
19. There are two bags. Bag X contains 5 white and 3 black balls and bag Y contains 3 white and 5 black balls. Two balls are drawn from bag A and put into bag B and then two balls are drawn from bag B. Find the probability that the balls drawn from bag B are white and black
20. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the sum of the numbers obtained is noted. If the result is a tail, a ticket is drawn from a pack of 15 tickets numbered 1, 2, 3, ..., 14, 15and the number on the ticket is noted. What is the probability that the noted number is either 12 or 13?
21. Bag A contains 1 white, 2 black and 3 red balls; bag B contains 2 white, 1 black and 1 red ball and bag C contains 4 white, 5 black and 3 red balls. A bag is chosen at random and two balls are drawn. What is the probability that the balls are white and red?

22. A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A containing 3 red and 2 white balls. If 2 or 3 turn up, a ball is picked up from bag B containing 3 red and 4 white balls. And if 4,5 or 6 turn up a ball is picked up from bag C containing 4 red and 5 white balls. The die is rolled, a bag is picked and a ball drawn. (a) what are the chances of drawing a red ball? (b) If the ball drawn is red, what are the chances that bag B was picked up.
23. A bag contains 4 balls. Two balls are drawn at random, and are found to be white. What is the probability that all balls are white? (Ans: 3/5)
24. 4 white and 3 black balls and 2 white and 2 black balls respectively. From bag A, two balls are drawn at random and then transferred to bag B. A ball is then drawn from bag B and is found to be black ball. What is the probability that the transferred balls were 1 white and 1 black? (Ans: 3/5)
25. A letter is known to have come either from "RANIGANJ" or "RANGANAPUR". On the envelope just two consecutive letters "AN" are visible. What is the probability that the letter is from (i) Raniganj (ii) Ranganapur ?
26. A letter is known to have come either from TATANAGAR or CALCUTTA. On the envelope just first two consecutive letters TA are visible. What is the probability that the letter has come from (i) Calcutta (ii) Tatanagar ?
27. Each of three identical has two drawers. In each drawer of the first box there is a gold watch. In one drawer of the third box there is a gold watch while in the other there is a silver box. In each drawer of the second box there is a silver watch. If we select a box at random, open one of the drawers and find it to contain a silver watch, what is the probability that the other drawer has the gold watch?
28. A fair coin is tossed until a head or five tails occur. If X denotes the number of tosses of the coin, find the mean and variance of X.
29. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times, find the probability distribution of number of tails. Also find the mean of the number of tails.
30. Two cards are drawn simultaneously (or, successively, without replacement) from a well shuffled pack of cards. Find the mean, variance and standard deviation of the number of kings.

31. A bag contains 5 white and 3 black balls. Four balls are drawn one at a time with replacement. Find the probability that the balls drawn are alternately of different colours.
32. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%? (Ans: 3 times)
33. An experiment succeeds twice as often as it fails. Find the probability that in the next 6 trials there will be at least 4 successes.
34. What is the probability of throwing doublets at least twice in four throws of a pair of dice?

Assignment No. 13
Probability

(Q1-Q5 are very short and short answer questions)

1. If $P(A)=\frac{1}{4}, P(B)=\frac{1}{2}, P(A\cap B)=\frac{1}{8}$, then find P(not A and not B).

2. If $P(A\cup B)=0.8$ and $P(A\cap B)=0.3$ then find $P(\bar{A})+P(\bar{B})$

3. A couple has 2 children. Find the probability that both are boys given that the older child is a boy.

4. Given that the events A and B are such that $P(A)=\frac{1}{2}$, $P(A\bigcup B)=\frac{3}{5}$ & $P(B)=K$, Then find "K" if A and B are independent.

5. If A and B are two independent events and $P(A)=0.3, P(B)=0.6$, then find $P(A\ \&\ notB)$

6. A car producing company knows from past experience that the probability of an order for cars will be ready for shipment on time is 0.85, and the probability that an order for cars will be ready for shipment and will be delivered on time is 0.75. What is the probability that an order for cars will be delivered on time given that it was ready for shipment on time?

7. A pair of dice is thrown and the product of the numbers is observed to be even. What is the probability that both dice have come up with even numbers?

8. A speaks truth in 55 percent cases and B speaks truth in 75 percent cases. Determine the percentage of cases in which they are likely to contradict each other in stating the same fact.

9. A bag contains 4 yellow and 5 red balls and another bag contains 6 yellow and 3 red balls. Two balls are drawn at random from the first bag and are transferred to the second bag. Then a ball is drawn from the second bag. Find the probability that it is yellow in colour.

10. Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be white and red. What is the probability that they came from Bag III.

11. There are two bags, one of which contains 3 black and 4 white balls, while the other contains 4 black and 3 white balls. A fair die is cast, if the face 1 or 3 turns up, a ball is

taken from the first bag, and if any other face turns up a ball is chosen from the second bag. Find the probability of choosing a black ball.

12. A bag contains 3 green and 7 white balls. Two balls are selected at random without replacement. If the second selected ball is given to be green what is the probability that the first selected ball is also green.
13. A letter is known to have come either from LONDON or CLIFTON. On the envelope just two consecutive letters ON are visible. What is the probability that the letter has come from (i) LONDON (ii) CLIFTON?
14. Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn successively without replacement. Also find the mean, variance and standard deviation of the distribution.
15. A bag contains 4 red and 5 black marbles. Find the probability distribution of number of red marbles in a random draw of three marbles. Also find the mean and standard deviation of the distribution.
16. In four throws of a pair of dice, what is the probability of throwing doublets (i) at least twice (ii) at most twice (iii) at least thrice (iv) not more than once.
17. If the probability of "success" in each trial of an experiment is $1/4$, then how many trials are necessary so that the probability of getting at least one success is greater than $2/3$?

Learning Outcomes:

Students will be able to

- appreciate use of multiplication theorem of probability
- recognize independent and dependent events
- identify conditional probability and apply properties of conditional probability
- apply theorem of total probability
- state and use Bayes' theorem
- define random variable of a given experiment
- find probability distribution of a random variable
- find mean and variance of a probability distribution
- identify Bernoulli trials
- find binomial distribution of the number of successes

Case Study Questions

Scan the QR Code and attempt the five Case Study questions

1. **Case Study 1**

2. **Case Study 2**

3. **Case Study 3**

4. **Case Study 4**

5. **Case Study 5**

विद्या च विमुच्यते
SANSKRITI
THE CIVIL SERVICES SCHOOL

Practice Assignment-I

Very Short and short Answer Type Questions

1. At the point $(2,1)$, find the slope of the curve $x^6 y^6 = 64$.

2. Find the derivative of $\sin^{-1}\left(x^3\right)$.

3. Evaluate $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

4. If "c" is a number that satisfies the conclusions of the Mean Value theorem for $x^3 - 2x^2$ on the interval $[0,2]$, find the value of "c".

5. If $f(x) = \sqrt{9-x};\ g(x) = x^3 + 1$, find $f \circ g(x)$.

6. If $f(x) = (x+1)e^x$, find the intervals in which the function is increasing.

7. Write the equation of the tangent to the curve $x^3 - 3x + 2$ at the point $(2,4)$.

8. Find the stationary points of the function $f(x) = (x-2)^{\frac{2}{3}}(2x+1)$

9. Find the maximum value of the function $f(x) = \sin 2x$ on the interval $\left[0, \frac{\pi}{2}\right]$.

10. If $f(x) = x^4$, defined from $R \to R$, is this function one – one?

11. If given that $f(x) = 16x^2 + 8x - 14$, is an invertible function, find its inverse.

12. Differentiate $\cos\left(x^x\right)$ with respect to x^x.

13. Find the slope of the tangent to the curve represented by $x = t^2 + 3t - 8;\ y = 2t^2 - 2t - 5$ at $(2,-1)$.

14. If $y = \tan^{-1}\frac{4x}{1+5x^2} - \tan^{-1}\frac{2-3x}{3+2x}$, show that $\frac{dy}{dx} = \frac{5}{1+25x^2}$.

15. Differentiate $\log x$ with respect to e^x.

16. Differentiate $\tan^{-1}\frac{2x}{1-x^2}$ with respect to $\sin^{-1}\frac{2x}{1+x^2}$.

17. If $y = e^{x+e^{x+e^{x+e^{x+\dots\infty}}}}$, prove that $\frac{dy}{dx} = \frac{y}{1-y}$.

18. If $y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x+....\infty}}}$, prove that $\frac{dy}{dx}=\frac{\sin x}{1-2y}$.

19. If $y=\sqrt{x}+\frac{1}{\sqrt{x}}$, show that $2x\frac{dy}{dx}+y=2\sqrt{x}$.

20. If $y=\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$, show that $\frac{dy}{dx}=0$.

21. Differentiate $\tan^{-1}\left(\frac{x^{\frac{1}{3}}+a^{\frac{1}{3}}}{1-x^{\frac{1}{3}}a^{\frac{1}{3}}}\right)$ with respect to "x"

22. If $y=\sin^2 x^2$, find $\frac{dy}{dx}$.

23. If $y=\sqrt{x+y}$, prove that $\frac{dy}{dx}=\frac{1}{2y-1}$.

24. Find $\frac{dy}{dx}$, if $x=a\log t; y=b\sin t$.

25. Find $\frac{dy}{dx}$, if $x=\sqrt{\sin 2\theta}; y=\sqrt{\cos 2\theta}$.

26. If $x=at^2, y=2at$ find $\frac{d^2y}{dx^2}$.

27. Show that the function $f(x)=2x+3$ is continuous at $x=-4$.

28. Show that the function $|x-4|$ is a continuous function.

29. Show that the function $f(x)=\begin{cases}\frac{x}{\sin 3x}, & x\neq 0\\ 3, & x=0\end{cases}$ is discontinuous at x=0

30. If the function $f(x)=\begin{cases}\frac{\sin^2 kx}{x^2}, & x\neq 0\\ 1, & x=0\end{cases}$, is continuous at x=0, find "k".

31. Show that the function $f(x)=\sin|x|$ is a continuous function.

32. Show that the function $f(x)=\frac{1}{x-5}$ is a continuous function.

33. If $\tan^{-1}3+\tan^{-1}x=\tan^{-1}8$, then find x.

34. Show that the function $f(x)=\sin^2 x+x^2-2x$ is continuous at x=0.

35. Evaluate a) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ b) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

36. Find the Principal value of $\cot^{-1}\left(-\sqrt{3}\right)$.

37. Simplify $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$.

38. Find the value of a) $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$ b) $\cos\left(\sec^{-1}x+\operatorname{cosec}^{-1}x\right), |x|\ge 1$

39. Find the value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$.

40. The function $f(x)=\begin{cases}\frac{\sin 3x}{x}, & x\neq 0\\ \frac{k}{2}, & x=0\end{cases}$ is continuous at "x = 0". Find "k"

41. Differentiate $\cos^{-1}\left(\frac{2x}{1+x^2}\right), -1<x<1$ with respect to "x"

42. Differentiate $\tan^{-1}\left(\sqrt{1+x^2}-x\right)$, $x\in R$ with respect to "x"

43. Differentiate with respect to "x": $\tan^{-1}\left(\frac{a+x}{1-ax}\right)$

44. Differentiate with respect to "x": $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

45. If $\sin y = x\sin(a+y)$, find $\frac{dy}{dx}$.

विद्यया अमृतमश्नुते SANSKRITI THE CIVIL SERVICES SCHOOL

Practice assignment -II

Very Short and short Answer Type Questions

Q1 Find the integrating factor for the following differential equation: $\frac{dx}{dy}-\frac{2x}{y}=3y^3-5y+1$

Q2 Show that the following differential equation is homogenous: $x\frac{dy}{dx}-y=\sqrt{x^2+y^2}$

Q3 If A, B are symmetric matrices and AB= BA, then show that AB is symmetric.

Q4 If A, B and AB are all symmetric matrices, then show that AB= BA.

Q5 If A, B are skew symmetric matrices and AB = BA, then show that AB is symmetric.

Q6 If A, B are square matrices of equal order and B is a skew symmetric matrix, then show that ABA' is also skew symmetric.

Q7 If a matrix is both symmetric and skew symmetric, then show that it is a null matrix.

Q8 What is the number of all possible matrices of order 3×3 with each entry 0 or 1?

Q9 If A, B are square matrices of equal order and B is a symmetric matrix, then show that $A'BA$ is also symmetric.

Q10 Give an example of two non-zero matrices A and B such that AB = O.

Q11 Give an example of two non-zero matrices A and B such that AB = O but BA ≠ O.

Q12 What is the order of AB + CB, where A, B and C are matrices of order $3\times 4, 4\times 2, 3\times 4$ respectively.

Q13 Give an example of symmetric and skew symmetric matrix.

Q14 If A= $\left[a_{ij}\right]$ is 3×3 matrix and $A_{ij}'s$ denote the cofactors of the corresponding elements $a_{ij}'s$, then write the value of $a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$.

Q15 If A= $\left[a_{ij}\right]$ is 3×3 matrix and $A_{ij}'s$ denote the cofactors of the corresponding elements $a_{ij}'s$, then write the value of $a_{21}A_{11}+a_{22}A_{12}+a_{23}A_{13}$.

Q16 If A= $\left[a_{ij}\right]$ is 3×3 matrix and $A_{ij}'s$ denote the cofactors of the corresponding elements $a_{ij}'s$, then write the value of $a_{11}A_{13}+a_{21}A_{23}+a_{31}A_{33}$.

Q17 If A is a square matrix of order 2 and $|A|=-5$, find the value of $|3A|$.

Q18 If A is a square matrix of order 3 and $|A|=-2$, find the value of $|5A|$.

Q19 If $x\in I$ and $\begin{vmatrix}2x & 3\\ -1 & x\end{vmatrix}=\begin{vmatrix}3 & 1\\ x & -1\end{vmatrix}$, find the value(s) of x.

Q20 Evaluate without expanding: $\begin{vmatrix} 2 & 2 & 2 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix}$

Q21 If A is a square matrix of order 3 such that $|adjA| = 100$, find $|A|$

Q22 If A, B and C are all non-zero square matrices of the same order , then find the condition on A such that AB = AC implies B = C.

Q23 If A is a skew symmetric matrix of order 3, then show that $|A| = 0$.

Q24 Examine whether the following system of equations is consistent : $2x - y = -2, 2y - z = -1, 3x - 5y = 3$.

Q25 Prove that the diagonal elements of a skew symmetric matrix are zero.

Q26 Find the values of x, y and z from the following equation: $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Q27 Evaluate without expanding: $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

Q28 If A=$\{$-1,1,3$\}$, then what is the number of relations on A?

Q29 Show that the function $f : N \to N$, defined by $f(x) = 2x - 1$ is not onto.

Q30 Is the function f defined by $f(x) = \begin{cases} 2x-1, x<0 \\ x+2, x \geq 0 \end{cases}$ continuous at x = 0.

Q31 A four digit number is formed using the digits 1, 2, 3, 5 with no repetition. Find the probability that the number is divisible by 5.

Q32 Find the rate of change of the area of a circle with respect to its radius when the radius is 5 cm.

Q33 If A is a matrix of order 3×4 then what should be the order of the matrix B such that $A'B$ and BA' are both defined?

Q34 Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

Q35 Show that $\cot^{-1} x + x$ is an increasing function on R.

Q36 Differentiate $\cot(x^{\cos x})$ w.r.t $x^{\cos x}$.

SANSKRITI THE CIVIL SERVICES SCHOOL

Q37 Find the maximum and minimum value of $2\sin x + 3\cos x$

Q38 If $y = \sqrt{\cos x + y}$, find $\frac{dy}{dx}$.

Q39 Find the value of x if $\begin{vmatrix} 2 & x \\ 3 & 7 \end{vmatrix} = \begin{vmatrix} 2 & 8 \\ -1 & 7 \end{vmatrix}$

Q40 Each side of an equilateral triangle is increasing at the rate of 8cm/hr. Find the rate of increase of its area when side is 2cm.

Q41 Find a, for which $f(x)$ = a(x + sin x) + a is increasing.

Q42 What is the approximate change in the volume V of a cube of side x cm caused by increasing the side by 2%?

Q43 The diameter of a circle is increasing at the rate of 1cm/sec. Find the rate of increase of its area when its radius is π .

Q44 If the tangent to the curve $x = at^2, y = 2at$ is perpendicular to x-axis, then find its point of contact.

Q45 Evaluate: $\tan^{-1}\left(x/y\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$

Q46 Differentiate $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ w.r.t. x.

Q47 If Rolle's theorem is applicable to $f(x) = e^x \sin x$ in $[0, \pi]$, then find the 'c' in Rolle's theorem.

Q48 If $\cos(x-y) = \log(x-y)$, then find $\frac{dy}{dx}$.

Q49 If the curve $ay + x^2 = 7$ and $x^3 = y$ cut orthogonally at (1, 1) then show that a = 6.

Q50 Write in the simplest form: $\tan^{-1}\left(\frac{2\sqrt{x}}{1-x}\right)$

THE CIVIL SERVICES SCHOOL

Practice Assignment-III

Very Short and short Answer Type Questions

Q1 If a line makes angles α, β, γ with the x, y, z axes respectively, find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

Q2 Evaluate, $\int_0^{1.5} [x]\, dx$ (where [x] is greatest integer function)

Q3 Evaluate, $\int_0^{1.5} [x^2]\, dx$ (where [x] is greatest integer function)

Q4 Write the order and degree of the differential equation, $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

Q5 If $f(1) = 4; f'(1) = 2$, find the value of the derivative of $\log f(e^x)$ w.r.t x at the point $x = 0$.

Q6 Let $f : R - \left\{-\frac{3}{5}\right\} \to R$ be a function defined as $f(x) = \frac{2x}{5x+3}$, find f^{-1} : Range of $f \to R - \left\{-\frac{3}{5}\right\}$

Q7 Let $f(x) = \sin x, g(x) = 2x$ and $h(x) = \cos x$, show that $f \circ g = g \circ (fh)$

Q8 If $y = f(x) = \frac{1-x}{1+x}$, show that $x = f(y)$

Q9 Let S=$\{1,2,3\}$ Find whether the function $f : S \to S$ defined as $f = \{(1,3),(3,2),(2,1)\}$ has inverse. If yes, find f^{-1}.

Q10 Let A = $\{0,3,5\}$, define a relation on A which is reflexive and transitive but not symmetric.

Q11 Is signum function $f : R \to R$, given by $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0 & , x = 0 \end{cases}$ an onto function? Justify your answer.

Q12 Let $f(x) = 1/x$ and $g(x) = 0$ be two real valued functions. Is $f \circ g$ defined? Justify.

Q13 Given that $f : R \to A$ given by $f(x) = \frac{x^2}{x^2+1}$ is a surjection , find A.

Q14 $f : A \to A$, where A=$[-1,1]$ given by $f(x) = x/3$. Is f bijective?

Q15 Check if the following functions are one – one, many – one, onto or into

a) $f: R \to R$; $f(x) = |x| + x$ b) $g: R \to R$; $g(x) = x^3$.

Q16 For any three vectors $\vec{a}, \vec{b}, \vec{c}$, write the value of $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$.

Q17 Find vector equation of line through points with position vectors $2\hat{i} - 3\hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

Q18 What should be the angle between vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$.

Q19 Write a value of $\int \frac{1 + \cot x}{x + \log \sin x} dx$

Q20 Show that the differential equation $2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$ is homogenous.

Q21 If $f'(x) = \frac{4}{x^2}$ and $f(1) = 6$, find $f(2)$

Q22 If $f(x) = e^x g(x), g(0) = 2$ and $g'(0)=1$, then find $f'(0)$.

Q23 What are the maximum and minimum values of 3sinx + 4 cosx .

Q24 Let f and g be differentiable functions satisfying

$g'(a) = 2, g(a) = b$ and $f \circ g = I$ (identity function), show that $f'(b) = \frac{1}{2}$.

Q25 If $y = \cos^{-1}\left(\frac{2\cos x - 3\sin x}{\sqrt{13}}\right)$, then show that $\frac{dy}{dx} = 1$.

Q26 Write a value of $\int (\cos(\log x) + \sin(\log x))\, dx$

Q27 If $f(x) = \frac{|x|}{x}, x \neq 0$, show that $|f(\alpha) - f(-\alpha)| = 2$, where $\alpha \neq 0$

Q28 If $f(x) = (a - x^n)^{1/n}$, then find $(f \circ f)(x)$

Q29 Write the order and degree of differential equation $x\frac{dy}{dx} + \frac{3}{\frac{dy}{dx}} = y^2$

Q30 Let $f: R \to R$ be a mapping defined by $f(x) = x^3 + 5$, find $f^{-1}: R \to R$

Q31 If $f(x) = \sin x$ and $g(x) = \cos x$, find $(2f)\left(\pi/2\right)$ and $(f - g)\left(\pi/2\right)$

Q32 Write a vector of magnitude 15 units in the direction of $2\hat{i} + 4j - 5k$

Q33 For any $\vec{r}$, find $(\vec{r}.\hat{i})\hat{i} + (\vec{r}.\hat{j})\hat{j} + (\vec{r}.\hat{k})\hat{k}$

Q34 If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with j , and an acute angle θ with k , then find the value of θ.

Q35 If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 3 and 2/3 respectively such that $\vec{a}\times\vec{b}$ is a unit vector, find the angle between $\vec{a}$ and $\vec{b}$.

Q36 Find $|\vec{x}|$ if $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=75$, where $\vec{a}$ is a unit vector.

Q37 Find a solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}$

Q38 Find the integrating factor of differential equation $\cos x\frac{dy}{dx}+y\sin x=1$

Q39 Write the order and degree of differential equation $5\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^3\right]^{3/2}$

Q40 Find the differential equation corresponding to $y=Ae^x+Be^{-x}$

Q41 Let $f:R\to R$ be given by $f(x)=x^2-3$. Find $f^{-1}:R\to R$.

Q42 Find the slope of the tangent to the curve $x=t^2+3t-8, y=2t^2-2t-5$ at the point (2,-1).

Q43 Write the position vector of a point dividing the line segment joining points A and B whose position vectors are $2\hat{i}-3\hat{j}+5\hat{k}$ and $3\hat{i}+2\hat{j}-\hat{k}$ in the ratio 1:4 externally.

Q44 If $\theta=\sin^{-1}(\sin(-600^\circ))$, then find one of the possible values of θ.

Q45 If $f:[2,\infty[\to X$ defined by $f(x)=4x-x^2$ is given to be invertible , then find X.

Q46 Evaluate: $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right\}$

Q47 If $y=\log\sqrt{\tan x}$, then find $\frac{dy}{dx}$.

Q48 If $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ then find $\frac{dy}{dx}$.

Q49 If $x=a\cos nt-b\sin nt$, then find $\frac{d^2x}{dt^2}$.

Q50 If the line y = x touches the curve $y=x^2+bx+c$ at (1, 1), then show that b = -1 and c = 1.

Q51 If the curves $y=ae^x$ and $y=be^{-x}$ cut orthogonally , then show that $ab=1$.

Q52 If $\tan^{-1}(\cot\theta)=2\theta$, then find a possible value of θ.

Q53 Evaluate $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

Q54 Find a unit vector parallel to the sum of $5\hat{i}-\hat{j}+2\hat{k}$ and $-\hat{i}+6\hat{j}+\hat{k}$

Q55 Find the vector projection of $3\hat{i}-\hat{j}+5\hat{k}$ on $-2\hat{i}+3\hat{j}+\hat{k}$

Q56 Find a vector which is equally inclined to the axes.

Q57 Find the value λ of if $22\hat{i}-3\hat{j}+5\hat{k}$ is perpendicular to $2\hat{i}+\lambda\hat{j}+\hat{k}$

Q58 If $\vec{a}$ and $\vec{b}$ are unit vectors such that $|\vec{a}+\vec{b}|=1$, then find $|\vec{a}-\vec{b}|$

Q59 If $\sin^{-1}x-\cos^{-1}x=\pi/6$, then find *x*.

SANSKRITI
THE CIVIL SERVICES SCHOOL

Practice Test-1

Relations and Functions

Q1. Give an example of a relation which is symmetric and reflexive but not transitive.

Q2. Check the injectivity of $f: Z \to Z$ given by $f(x) = x^2$

Q3. If $f = \{(1,3),(2,7),(8,6)\}$ and $g = \{(7,11),(6,0),(3,5)\}$, find gof.

Q4. If f :R→R defined by $f(x) = 3x - 4$ is invertible then write $f^{-1}(x)$.

Q5. Let f :R→R be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then find $fof(x)$

Q6. Consider $f: R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible. Find the inverse of f.

Q7. Let R be a relation on the set A of ordered pairs of positive integers defined by $(x, y) R (u, v)$ if and only if $xy = uv$. Show that R is an equivalence relation.

Q8. Show that the function $f: R \to R$ defined by $f(x) = \dfrac{3x+1}{2}$ is one- one and onto. Hence find the inverse of the function.

Q9. Show that the function $f: A \to B$ defined as $f(x) = \dfrac{3x+4}{5x-7}$, where $A = R - \left\{\dfrac{7}{5}\right\}, B = R - \left\{\dfrac{3}{5}\right\}$ is invertible and hence find f^{-1}.

Q10. Show that the relation R on the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a,b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1 i.e., equivalence class [1].

SANSKRITI
THE CIVIL SERVICES SCHOOL

Practice Test-2

Inverse Trigonometric Functions

Q1. Write the value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$.

Q2. Write the value of $2\cos^{-1}\frac{1}{2}+3\sin^{-1}\frac{1}{2}$

Q3. Evaluate $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}\left(\cos\left(\frac{-\pi}{3}\right)\right)$

Q4. Solve for x: $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$

Q5. Evaluate : $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

Q6. Evaluate : $\cot^{-1}\left[2\sin\left(2\cos^{-1}\frac{1}{2}\right)\right]$

Q7. Solve for x : $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

Q8. Prove that : $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2}$, $x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

Q9. Prove that: $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$

Q10. Prove that : $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1+x}}{\sqrt{1+x}+\sqrt{1+x}}\right)=\left(\frac{\pi}{2}-\frac{1}{2}\cos^{-1}x\right)$

Q11. Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

Q12. Show that : $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$

SANSKRITI THE CIVIL SERVICES SCHOOL

Practice Test 3

Matrices and Determinants

Q1. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, find the value of x.

Q2. Find B if A= $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and 2A + B= $\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Q3. If A= $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^4 = \lambda A$, find the value of λ

Q4. If A is a square matrix of order 3 and $|A| = -4$, find the value of $|4A|$.

Q5. Express $\begin{bmatrix} 2 & 5 & -9 \\ 3 & 0 & -1 \\ 4 & 2 & 5 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrices.

Q6. If $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = O$.

Q7. Using elementary transformations find the inverse of $\begin{bmatrix} 3 & 1 \\ 6 & 4 \end{bmatrix}$.

Q8. By using properties of determinants show that:

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & xz \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

Q9. By using properties of determinants show that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

Q10. An amount of Rs 600 crores is spent by the government in three schemes. Scheme A is for saving girl child. Scheme B is for saving of newlywed girls from death due to dowry. Scheme C is planning for good health for senior citizen. Twice the amount spent on Scheme C together with amount spent on Scheme A is Rs 700 crores. And three times the amount spent on Scheme A together with amount spent on Scheme B and Scheme C is Rs 1200 crores. Find the amount spent on each Scheme using matrices? What is the importance of saving girl child? Suggest any one measure that you will do to save girl child.

SANSKRITI THE CIVIL SERVICES SCHOOL

Practice Test 4
Differentiation

Q1. If $f(x)=\begin{cases}\dfrac{x^2-2x-3}{x+1}, & x\neq -1\\ \lambda, & x=-1\end{cases}$ is continuous at x= -1, find the value of λ.

Q2. Differentiate $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ w.r.t. $\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

Q3. If $\log\left(\sqrt{1+x^2}-x\right)=y\sqrt{1+x^2}$ show that $(1+x^2)\dfrac{dy}{dx}+xy+1=0$.

Q4. Find the value of "a " for which the function defined as

$$f(x)=\begin{cases}a\sin\dfrac{\pi}{2}(x+1), x\leq 0\\ \dfrac{\tan x-\sin x}{x^3}, x>0\end{cases}$$ is continous at x = 0

Q5. Differentiate $x^{x\cos x}+\dfrac{x^2+1}{x^2-1}$ w.r.t x .

Q6. If $x^y=y^x$, show that $\dfrac{dy}{dx}=\dfrac{y(x\log y-y)}{x(y\log x-x)}$

Q7. Find $\dfrac{dy}{dx}$ if $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$.

Q8. If $\cos^{-1}\left[\dfrac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right]$ find $\dfrac{dy}{dx}$.

Q9. If $x=a(\cos t+t\sin t)$ and $y=b(\sin t-t\cos t)$, find $\dfrac{d^2y}{dx^2}$ at $t=\dfrac{\pi}{3}$.

Q10. Verify Lagrange's Mean Value Theorem for $f(x)=x(x-1)(x-2)$ in $\left[0,\dfrac{1}{2}\right]$.

Q11. If $x=\sin t$ and $y=\sin pt$,prove that $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+p^2y=0$.

Q12. Differentiate $\cos^{-1}\left[\dfrac{\sqrt{1-x^2}+x}{\sqrt{2}}\right]$ w.r.t $\tan^{-1}\left[\dfrac{x}{\sqrt{1-x^2}+1}\right]$.

Q13. If $\cos y=x\cos(a+y)$,prove that $\dfrac{dy}{dx}=\dfrac{\cos^2(a+y)}{\sin a}$.

Q14. If $y=\left(\tan^{-1}x\right)^2$,prove that $(1+x^2)^2\dfrac{d^2y}{dx^2}+2x(1+x^2)\dfrac{dy}{dx}=2$.

Practice Test 5

Application of Derivatives

Q1. Use differential to approximate $\sqrt{0.0037}$.

Q2. Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 1%.

Q3. Find the intervals in which $(x-1)^3(x-2)^2$ is increasing and decreasing.

Q4. Find the equation of tangent and normal to the curve $x = a\sin^3 t, y = b\cos^3 t$ at the point 't".

Q5. Find equation(s) of tangent drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from the point (1,2).

Q6. Find the intervals in which $f(x) = 2\log(x-2) - x^2 + 4x + 1$ is increasing and decreasing.

Q7. Find all points of local maxima and minima and corresponding maximum and minimum values of the function $f(x) = \frac{-3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$.

Q8. Find points of local maxima and minima of $f(x) = \sin x + \frac{1}{2}\cos 2x$ where $0 \le x \le \frac{\pi}{2}$. Also find the absolute maximum and minimum values of the function.

Q9. Water is dripping out from a conical funnel at a uniform rate of 4 $cm^3/\sec$ through a tiny hole at the vertex in the bottom. When the slant height of water is 3 cm, find the rate of decrease of the slant height of the water cone given that the vertical angle of the funnel is 120°.

Q10. Sand is being poured into a conical pile at a constant rate of 50 cm^3/sec such that the height of the cone is always one half of the radius of the base. How fast is the height of the pile increasing when the sand is 5cm deep?

Q11. If the sum of the lengths of the hypotenuse and a side of a right triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

Q12. Show that the volume of the largest cone that can be inscribed in a sphere of radius 'r' is $\frac{8}{27}$ of the volume of the sphere.

Q13. An open box with a square base is to be made out of a given quantity of cardboard of area c^2 sq. units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cu. units.

SANSKRITI THE CIVIL SERVICES SCHOOL

Practice Test 6

Integration

Evaluate the following:

Q1. $\int \frac{x^2-3x}{(x-1)(x-2)}dx$

Q2. $\int \frac{\sin x}{\sin 4x}dx$

Q3. $\int e^x \cos^2 x \, dx$

Q4. $\int \frac{2\sin 2x-\cos x}{6-4\cos^2 x-4\sin x}dx$

Q5. $\int_0^{3/2} |x\cos \pi x| dx$

Q6. $\int \frac{x-\sin x}{1-\cos x}dx$

Q7. $\int \frac{dx}{5+7\cos x+\sin x}$

Q8. $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}dx$

Q9. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}dx$

Q10. $\int_0^1 (3x^2+2x)dx$ as a limit of sums.

Q11. $\int \frac{dx}{\sin x(5-4\cos x)}$

विद्यया च विमुच्यते
SANSKRITI
THE CIVIL SERVICES SCHOOL

Practice Test 7

Application of Integration and Differential Equations

Q1. Find the differential equation of all circles touching the x axis at the origin.

Q2. Find the differential equation of all circles in the first quadrant which touch the coordinate axes.

Q3. Solve: $e^{\frac{dy}{dx}} = x+1$, $y(0)=5$

Q4. Solve: $x(xdy - ydx) = ydx$

Q5. Solve the differential equation: $(x+y)^2 \frac{dy}{dx} = a^2$

Q6. Solve the differential equation: $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$

Q7. Solve the differential equation: $(1+y^2)dx = (\tan^{-1} y - x)dy$

Q8. Find the particular solution of the equation: $ye^y dx = (y^3 + 2xe^y)dy$, $y(0)=1$

Q9. Find the area of the region $\{(x,y): y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$

Q10. Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and x=0.

Q11. Find area of the region common to $x^2 + y^2 = 16$ and $6y = x^2$

SANSKRITI
THE CIVIL SERVICES SCHOOL

Practice Test 8

Vectors and 3- Dimensional Geometry

Q1. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$; find a vector of magnitude 6 units which is parallel to $2\vec{a} - \vec{b} + 3\vec{c}$.

Q2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors of magnitude 3, 4, 5 respectively such that each is perpendicular to the sum of the other two, prove that $\left|\vec{a} + \vec{b} + \vec{c}\right| = 5\sqrt{2}$.

Q3. Find the equation of the plane passing through the points (3, 2, 1) and (0, 1, 7) and parallel to the line $\frac{x-2}{1} = \frac{y+1}{-1} = \frac{1-z}{-1}$

Q4. Find equation of the perpendicular drawn from the point (2, 4, -1) to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

Q5. The dot products of a vector with the vectors $\hat{i} - 3\hat{k}$, $\hat{i} - 2\hat{k}$ and $\hat{i} + \hat{j} + 4\hat{k}$ are 0,5 and 8 respectively. Find the vector.

Q6. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{a} \cdot \hat{b} = \hat{a} \cdot \hat{c} = 0$ and angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$ prove that, $\hat{a} = \pm 2\left(\hat{b} \times \hat{c}\right)$.

Q7. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = 3\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - \hat{k}$; find a vector $\vec{d}$ which is perpendicular to both $\hat{a}$ and $\hat{b}$ and $\vec{c} \cdot \vec{d} = 1$.

Q8. Find the coordinates of the point where the line through (3, -4, -5) and (2, -3, 1) crosses the plane determined by the points (1, 2, 3), (2, 2, 1) and (-1, 3, 6).

Q9. Find the equation of the plane through the intersection of the planes $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$, $\vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$.

Q10. Find the distance of the point (-1, -5, -10) from the point of intersection of the line $\vec{r} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and plane $\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) - 5 = 0$.

Q11. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (3,2,1) from the plane $2x - y + z + 1 = 0$. Also find the image of the point in the plane.

Practice Test 9

Probability

Q1. A bag contains 4 red and 4 black balls. Another bag contains 2 red and 6 black balls. One of the two bags is selected and a ball is drawn. If it is found to be red, find the probability the second bag was chosen.

Q2. The probability of A, B and C solving a problem independently is ½, ¾ and 1/3. If all of them try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly two of them will solve (iii) at most two of them solve.

Q3. There are three urns A, B and C. A contains 10red and 4 green marbles. B contains 9 red and 5 green marbles. C contains 8 red and 6 green marbles. One ball is drawn from each of these urns. What is the probability that, out of these three balls drawn, two are red and one is green?

Q4. Rohan and Sid throw a die alternatively till one of them gets a 6 and wins the game. Find the probability of Sid winning the game if Rohan starts the game.

Q5. There are two bags. The first bag contains 5 white and 6 black marbles. The second bag contains 4 white and 7 black marbles. Two balls are drawn from the first bag and without noticing their colour, are put into the second bag. Then two balls are drawn from the second bag. What is the probability that the balls drawn are white?

Q6. A man is known to speak the truth 3 out of 5 times. He throws a die and reports that a 4 has occurred. What is the probability that a 4 has actually occurred?

Q7. A card from a deck of 52 cards is lost. From the remaining cards two cards are drawn and found to be diamonds. What is the probability that a spade card is lost?

Q8. The probability of a man hitting a target is ½. How many times must he fire so that the probability of hitting the target at least once is more than 90%/

Q9. Find the probability distribution of number of sixes in three tosses of a die.

Q10. From a lot of 10 items containing 3 defective items, a sample of 4 items is drawn. Find the probability distribution of X, where X denotes the number of defective items drawn. Also find the mean and variance of the distribution.

Q11. In a bulb factory, machines A, B and C manufacture 60%, 30 % and 10 % bulbs respectively. 1 %, 2 % and 3% of the bulbs produce respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and is found to be defective. Find the probability that the bulb was produced by the machine A.

Academic Session: 2020-21

Pre-Board Examination I
Mathematics

Class XII

Time allowed - 3 hours **Maximum Marks – 80**

General Instructions:

- **This question paper has 7 pages containing two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.**
- **Both Part A and Part B have choices.**
  Part – A:
- **It consists of two sections- I and II.**
- **Section I comprises of 16 very short answer type questions. Internal choice is provided in 5 questions of Section I. You have to attempt only one of the alternatives in all such questions.**
- **Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.**
  Part – B:
- **It consists of three sections- III, IV and V.**
- **Section III comprises of 10 questions of 2 marks each.**
- **Section IV comprises of 7 questions of 3 marks each.**
- **Section V comprises of 3 questions of 5 marks each.**
- **Internal choice is provided in 4 questions of Section III, 3 questions of Section IV and 2 questions of Section V. You have to attempt only one of the alternatives in all such questions.**

Part A

Section-I

Question numbers 1 to 16 carry one mark each.

Q1. Write the vector equation of the line $x = 5, \frac{y+4}{7} = \frac{z-6}{2}$

Q2. What is the scalar projection of $\vec{a} = -2\hat{\imath} + \hat{\jmath} - 5\hat{k}$ on $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$?

Q3. For what values of x and y is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & y & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix?

OR

Given that A is a square matrix of order 3 and $|A| = -2$. Find $|adjA|$

Q4. Find the direction cosines of the vector $\overrightarrow{PQ}$ if the position vectors of P and Q are respectively $\hat{\imath} + \hat{\jmath} - \hat{k}$ and $2\hat{\imath} + \hat{\jmath} - \hat{k}$

Q5. Find the unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath}$

Q6. If $\vec{a}$ is any non-zero vector, then represent $(\vec{a}.\hat{\imath})\hat{\imath} + (\vec{a}.\hat{\jmath})\hat{\jmath} + (\vec{a}.\hat{k})\hat{k}$ purely in terms of $\vec{a}$

Q7. Find the matrix A if $\begin{bmatrix} -2 & -1 \\ -1 & -1 \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

Q8. If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and A_{ij} represents the cofactor of a_{ij}, then what is the value of $a_{11}A_{13} + a_{21}A_{23} + a_{31}A_{33}$?

Q9. Determine whether or not the relation R in the set **R** of real numbers defined as $R = \{(a, b): \sqrt{a} = b\}$ is a function? Justify your answer.

OR

Write the equivalence relation on the set $A = \{1,2,3\}$ having least number of elements in it.

Q10. What is the value of $cos^{-1}\left(cos\frac{2\pi}{3}\right) + sin^{-1}\left(sin\frac{2\pi}{3}\right)$?

OR

Find the value of $tan^{-1}[2\sin\left(2cos^{-1}\frac{\sqrt{3}}{2}\right)]$

Q11. Prove that the function $f: \mathbf{R} \to \mathbf{R}$ defined by $f(x) = |x|$ is not surjective.

Q12. Find $\int \frac{xe^x}{(x+1)^2} dx$

OR

Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^{13}x dx$

Q13. Write the I.F. of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$

OR

Find the sum of the order and the degree of the differential equation

$(\frac{d^3y}{dx^3})^2 + (\frac{dy}{dx})^4 + 1 = 0$

Q14. Using integration find the area of the region bounded by the curve $y = x|x|$, the lines $x = 0$ and $y = 1$

Q15. Let E and F be two events such that $P(E) = 0.3, P(E \cup F) = 0.4$ and $P(F) = x$. Then find the value of x such that E and F are independent.

Q16. Find the probability that the larger of the two integers selected at random from first six positive integers is 2.

Section-II

Both the case study-based questions 17 and 18 are compulsory. Attempt any 4 sub-parts from each question. Choose the correct option in each sub-part. Each sub-part carries 1 mark.

Q17. An ant is travelling along a wall tracing a path given by the function

$f(x) = sinx + \frac{1}{2}cos2x, x \in [0, \pi]$

From the given case study answer the following questions:

(i) Find the critical points of the function:

$(a)\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ $(b)\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}$ $(c)\frac{\pi}{6}, \frac{\pi}{4}, \pi$ $(d) 0, \frac{\pi}{2}, \pi$

(ii) Find the interval(s) in which the function is strictly increasing:

$(a)\left(\frac{\pi}{6}, \frac{\pi}{2}\right), \left(\frac{5\pi}{6}, \frac{\pi}{2}\right]$ $(b)\left[0, \frac{\pi}{6}\right), \left(\frac{\pi}{2}, \frac{5\pi}{6}\right)$ $(c)\left[0, \frac{\pi}{6}\right), \left(\frac{\pi}{4}, \pi\right]$ $(d)[0, \frac{\pi}{2})$

(iii) Find the interval(s) in which the function is strictly decreasing:

$(a)\left(\frac{\pi}{6},\frac{\pi}{2}\right),\left(\frac{5\pi}{6},\pi\right]$ $(b)\left[0,\frac{\pi}{6}\right),\left(\frac{\pi}{2},\frac{5\pi}{6}\right)$ $(c)\left[0,\frac{\pi}{6}\right),\left(\frac{\pi}{4},\pi\right]$ $(d)[0,\frac{\pi}{2})$

(iv) Find the maximum height attained by the ant (from the x-axis):

$(a) 1\ unit$ $(b)\frac{1}{\sqrt{2}}\ unit$ $(c)\ \frac{1}{2}\ unit$ $(d)\frac{3}{4}\ unit$

(v) Find the minimum height attained by the ant (from the x-axis):

$(a) 1\ unit$ $(b)\frac{1}{\sqrt{2}}\ unit$ $(c)\ \frac{1}{2}\ unit$ $(d)\frac{3}{4}\ unit$

Q18. There are three categories of students in a class of 40 students:

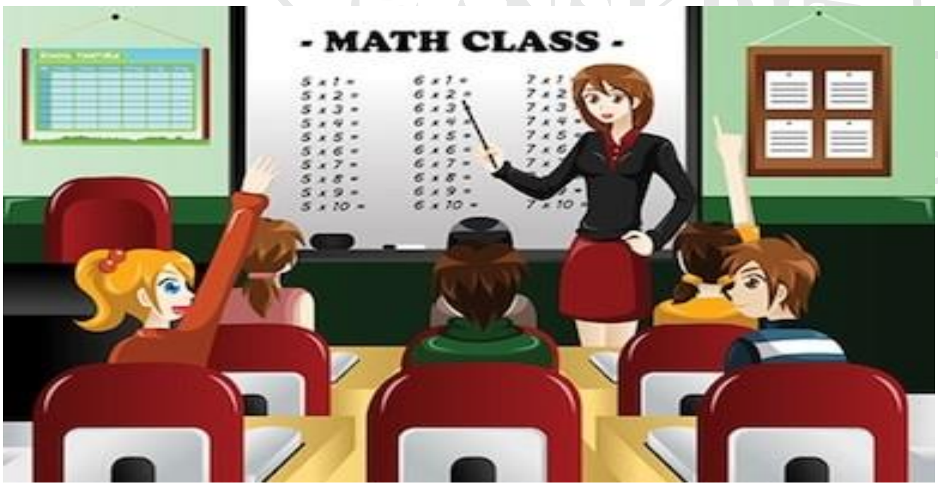
A: Very hard-working students

B: Regular but not so hard-working

C: Careless and irregular

It is also found that the probability of students of category A, category B and category C, unable to get good marks in the final examination are 0.002, 0.02, 0.80 respectively. It is known that 10 students are in category A, 20 in category B and rest in category C.

Based on the above information answer the following questions:



(i) If a student selected at random was found to be the one who could not get good marks in the examination. What is the probability that the student is of category C?

$(a)\ \frac{400}{421}$ $(b)\frac{1}{421}$ $(c)\ \frac{20}{421}$ $(d)\frac{120}{441}$

(ii) If a student selected at random was found to be the one who could not get good marks in the examination. What is the probability that the student is of category A or B?

$(a)\ \frac{100}{421}$ $(b)\frac{21}{421}$ $(c)\ \frac{20}{421}$ $(d)\frac{1}{421}$

(iii) If a student selected at random was found to be the one who could not get good marks in the examination. What is the probability that the student is of category A?

$(a)\ \frac{400}{421}$ $(b)\frac{1}{421}$ $(c)\ \frac{20}{421}$ $(d)\frac{120}{441}$

(iv) If a student selected at random was found to be the one who could not get good marks in the examination. What is the probability that the student is not of category A?

$(a)\ \frac{100}{421}$ $(b)\frac{21}{421}$ $(c)\ \frac{420}{421}$ $(d)\frac{120}{421}$

(v) What is the probability that the student selected at random could not get good marks in the examination?

$(a)\ \frac{421}{2000}$ $(b)\ \frac{21}{2000}$ $(c)\ \frac{20}{2000}$ $(d)\frac{1}{2000}$

Part B

Section-III

Q19. Evaluate $\sin\{2cos^{-1}\left(-\frac{3}{5}\right)\}$

Q20. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Then verify that $A(adjA) = |A|I$

OR

If a matrix A is skew-symmetric, then prove that the matrix B'AB is also skew-symmetric, where B is a square matrix of suitable order so that the matrix B'AB is defined.

Q21. Find the value of a so that the function f is continuous at $x = 0$

$$f(x) = \begin{cases} \frac{tanx - sinx}{x^3} & x \neq 0 \\ a, & x = 0 \end{cases}$$

Q22. The equation of the tangent at (2, 3) to the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b

Q23. Find $\int \frac{1}{\sqrt{(x-1)(x-2)}} dx$

OR

Evaluate $\int_0^{2\pi} \frac{1}{1+e^{sinx}} dx$

Q24. Using integration, find the area of the region $\{(x, y): x^2 \leq y \leq x\}$

Q25. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$

OR

Solve the differential equation $sec^2x\ tany\ dx + sec^2y\ tanx\ dy = 0$

Q26. Show that $\left|\vec{a} \times \vec{b}\right|^2 = |\vec{a}|^2\left|\vec{b}\right|^2 - (\vec{a}.\vec{b})^2$

Q27. Find the foot of the perpendicular from the point (0, -1, 2) upon the plane $2x + 4y - z + 2 = 0$

OR

Find the distance between the lines $\vec{r} = 2\hat{\imath} + \hat{k} + t(\hat{\imath} - \hat{\jmath} + 2\hat{k})$ and $\vec{r} = \hat{\imath} + \hat{\jmath} + \mu(\hat{\imath} - \hat{\jmath} + 2\hat{k})$

Q28. A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of the cases are they likely to contradict each other

SANSKRITI THE CIVIL SERVICES SCHOOL

Section-IV

Q29. Show that the relation R in the set **R** of real numbers defined as $R = \{(a, b): a \leq b^2\}$ is neither symmetric nor transitive.

Q30. Find $\frac{dy}{dx}$, if $x^y + y^x = 2$

OR

If $x = a\,sec^3\theta, y = a\,tan^3\theta$, then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$

Q31. Prove that the function $f(x) = |x - 2|$ is not differentiable at $x = 2$

Q32. Find the points of local maximum/local minimum, if any, of the function

$f(x) = sin^4x + cos^4x, x \in (0, \frac{\pi}{2})$

Q33. Find $\int \frac{1}{x(x^5-1)} dx$

OR

Find $\int \frac{1}{2+cosx} dx$

Q34. Using integration, find the area of the minor segment of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{2}$

OR

Using integration, find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, the minor axis and one of the latus rectums of the ellipse.

Q35. Solve the differential equation $x\,dy + (y - x^3)dx = 0, y(1) = 2$

Section-V

Q36. Using matrix method, solve the following system of equations:

$x + 2y + z = 7$

$x + 3z = 11$

$2x - 3y = 1$

OR

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to

solve the following system of equations:

$x - y + z = 4$

$x - 2y - 2z = 9$

$2x + y + 3z = 1$

Q37. Find the equations of the planes passing through the line of intersection of the planes

$\vec{r}.(2\hat{\imath} + 6\hat{\jmath}) + 12 = 0$ and $\vec{r}.(3\hat{\imath} - \hat{\jmath} + 4\hat{k}) = 0$ which are at a unit distance from the origin

OR

Find the distance of the point (1, -5, 9) from the plane $\vec{r}.(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$ measured along the line $x = y = z$

Q38. Solve the following linear programming problem (L.P.P.) graphically.

Minimize $Z = 20x + 10y$ subject to the constraints

$x + 2y \leq 40$

$3x + y \geq 30$

$4x + 3y \geq 60$

$x \geq 0$

$y \geq 0$

Academic Session: 2020-21

Pre-Board II Examination

Mathematics

Class XII

Time allowed - 3 hours **Maximum Marks – 80**

General Instructions:

- **This question paper has 7 pages containing two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.**
- **Both Part A and Part B have choices.**
  Part – A:
- **It consists of two sections- I and II.**
- **Section I comprises of 16 very short answer type questions. Internal choice is provided in 5 questions of Section I. You have to attempt only one of the alternatives in all such questions.**
- **Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.**
  Part – B:
- **It consists of three sections- III, IV and V.**
- **Section III comprises of 10 questions of 2 marks each.**
- **Section IV comprises of 7 questions of 3 marks each.**
- **Section V comprises of 3 questions of 5 marks each.**
- **Internal choice is provided in 3 questions of Section III, 4 questions of Section IV and 2 questions of Section V. You have to attempt only one of the alternatives in all such questions.**

Part A

Section-I

Question numbers 1 to 16 carry one mark each.

SANSKRITI THE CIVIL SERVICES SCHOOL

Q1. Find the length of the perpendicular drawn from the origin to the plane

$2x-3y+6z+21=0$.

Q2. Find the direction cosines of the line joining the points $P(4,3,-5)$ and $Q(-2,1,-8)$.

Q3. Prove that the diagonal elements of a skew symmetric matrix are all zero.

OR

Given a square matrix A of order 3×3 such that $|A|=4$, find the value of $|A\ adjA|$.

Q4. If $|\vec{a}|=2, |\vec{b}|=5$ and $|\vec{a}\times\vec{b}|=8$, find $\vec{a}.\vec{b}$

Q5. Find the area of a parallelogram whose adjacent sides are given by the vectors

$\vec{a} = \hat{j} - \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{j}$.

Q6. Find a vector in the direction of vector $2\hat{\imath} - \hat{j} + 2\hat{k}$ and of magnitude 3 units.

Q7. Find the value of x for which the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular.

Q8. Find the order of the matrix $\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}^T$.

Q9. Let A = $\{2,3,4,5\}$ .Define a relation on A which is reflexive and symmetric but not transitive.

Q10. If E_i is an equivalence class with respect to the relation R defined on a set A and i=1,2,3,4,5. If $x \in E_i$ and $y \in E_j, i \neq j$, then can we say that x and y are related to each other with respect to relation R?

OR

If R is an equivalence relation defined on set $A = \{1,2,3, \dots ,10\}$ as $R = \{(a, b): |a - b|\ is\ a\ multiple\ of\ 3\}$. Write the equivalence class of 1.

Q11. Show that $f: R \rightarrow R$ defined as $f(x) = sinx$ is not bijective.

OR

Consider a function $f : A \rightarrow A$, where A=$[-1,1]$ given by $f(x) = x/3$. Is f bijective?

Q12. Find $\int \frac{1}{\sqrt{4x^2-1}} dx$

OR

Evaluate: $\int_0^{1.5} [x] dx$

Q13. What should be the value of n such that the differential equation:

$x^n \frac{dy}{dx} = y(logy - logx + 1)$ is homogenous.

OR

Find the order and degree (if defined) of the differential equation:

$\frac{d^2y}{dx^2} = \left\{y + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{4}}$.

Q14. Using integration find the area of the region bounded by the line $y = 2x$, the y-axis and the line $y = 3$.

Q15. Assume that each child born is equally likely to be a girl or a boy. If a family has two children, then what is the conditional probability that both are girls, given that the youngest is a girl.

Q16. Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = p, P(A \cup B) = \frac{3}{5}$.

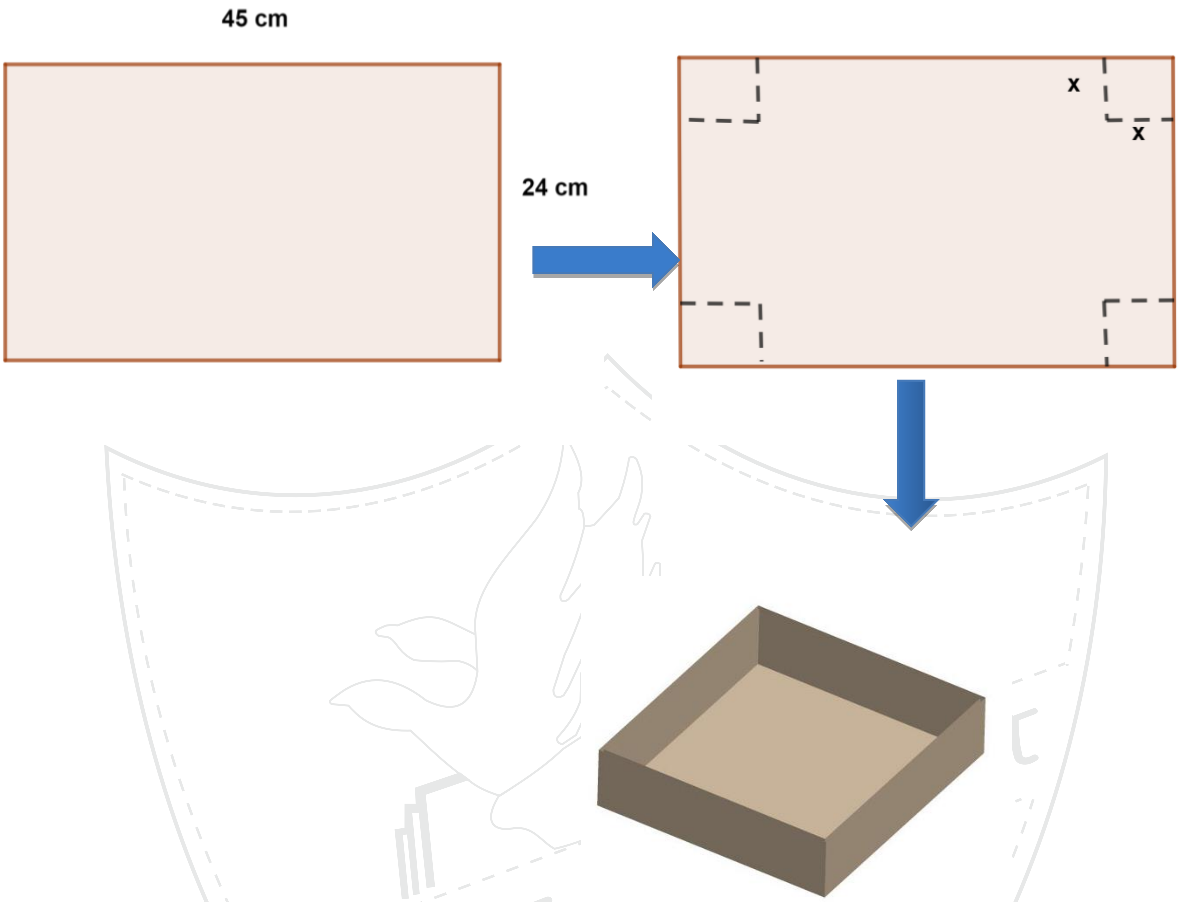
Find p if A and B are independent events.

Section-II

Both the case study-based questions 17 and 18 are compulsory. Attempt any 4 sub-parts from each question. Choose the correct option in each sub-part. Each sub-part carries 1 mark.

Q17. Three friends A, B and C are given a rectangular sheet of sides 45 cm and 24 cm each.

They are asked to work independently and form an open box by cutting the squares of Equal length from all the four corners as shown and folding up the flaps, they want to check the volume of boxes so formed.



From the given case study answer the following questions:

(i) If a square of side x cm is cut from all corners, then length, breadth and height of box are:

$(a)\ l = (45 - 2x)cm, b = (24 - 2x)cm, h = x\ cm$

(b) $l = (45 - x)cm, b = (24 - x)cm, h = x\ cm$

$(c) l = (45 - x)cm, b = (24 - x)cm,\ \ h = 2x\ cm$

$(d) l = (45 - 2x)cm, b = (24 - 2x)cm, h = 2x\ cm$

(ii) Volume of the box will be:

$(a)(45 - 2x)x\ cm^3$ $(b)(45 - x)(24 - x)(x)cm^3$

$(c)(45 - 2x)(24 - 2x)(x)cm^3$ $(d)(45 - x)(24 - x)(2x)cm^3$

SANSKRITI THE CIVIL SERVICES SCHOOL

(iii) The three friends are cutting the square of different sizes from the corners then:

(a) There is no change in volume
(b) Greater the side of the square, maximum will be the volume.
(c) Lesser the side of the square, maximum will be the volume
(d) Volume will be maximum if we cut square of a particular length, based on dimensions of given sheet.

(iv) Volume of the box will be maximum for x equal to:

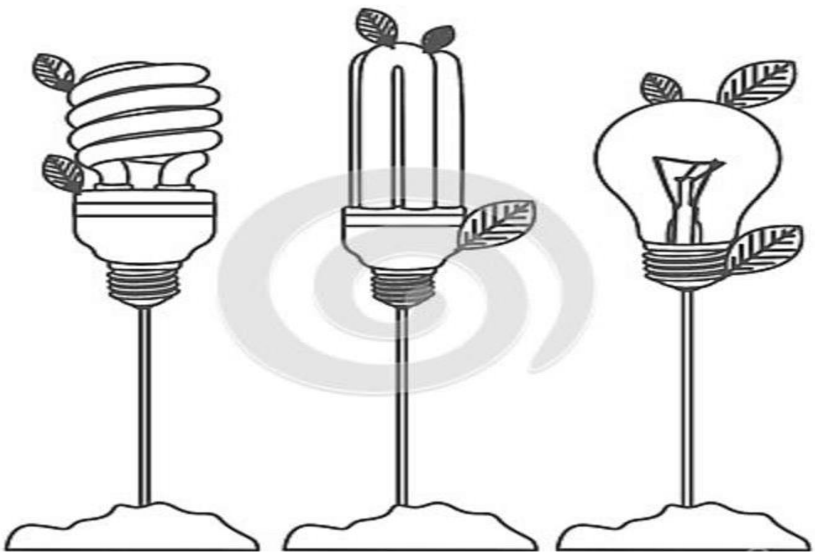
$(a) 18\ cm$ $(b) 5\ cm$ $(c)\ 16\ cm$ $(d) 7\ cm$

(v) Maximum volume of box is:

$(a) 5600\ cm^3$ $(b) 4000\ cm^3$ $(c)\ 3500\ cm^3$ $(d) 2450\ cm^3$

Q18. A company producing electric bulbs has factories at three locations E_1, E_2 *and* E_3 and company got a bulk order of producing electric bulbs. The capacities at locations E_1 *and* E_3 are same and at location E_2 is double that of E_1. Also it is known that 4% of bulbs produced at E_1 and E_2 are defective and 5% produced at E_3 are defective.

Based on the above information answer the following questions:



(i) What is the probability of production capacity of E_3 ?

$(a)\ \frac{1}{2}$ $(b)\ \frac{1}{4}$ $(c)\ \frac{1}{3}$ $(d)\ \frac{1}{5}$

(ii) What is conditional probability of producing defective bulb by E_2?

(a) 96% (b) 4 % (c) 5% (d) 95%

(iii) What is the probability of a defective bulb?

$(a)\ \frac{13}{400}$ (b) 0.01 (c) 0.02 $(d)\ \frac{17}{400}$

(iv) If a defective bulb has been produced, what is the probability that it is produced at location E_2?

$(a)\ \frac{1}{17}$ $(b)\ \frac{2}{13}$ $(c)\ \frac{4}{13}$ $(d)\ \frac{8}{17}$

(v) Which location produces least defective bulbs?

$(a)\ E_1$ $(b)\ E_2$ $(c)\ E_3$ (d) *none of these*

Part B

Section-III

Q19. Simplify the following: $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$

Q20. Express $A=\begin{bmatrix}3 & 5\\7 & 9\end{bmatrix}$ as sum of a symmetric and a skew symmetric matrix.

OR

Find the inverse of the matrix $A=\begin{bmatrix}a & b\\c & \frac{1+bc}{a}\end{bmatrix}$.

Q21. For what value of "k" the function $f(x)=\begin{cases}\dfrac{\sqrt{5x+2}-\sqrt{4x+4}}{x-2}; & x\neq 2\\ k & x=2\end{cases}$ is continuous at $x=2$

Q22. Find the point(s) on the curve $y = x^3 + 7x$ at which the normal line has the equation

$y = x + 20.$

Q23. Find: $\int \frac{1}{sinx+\sqrt{3}cosx}dx$

OR

Evaluate: $\int_{e}^{e^2}\frac{1}{x\log x}dx$

Q24. Using integration, find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9} = 1.$

Q25. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that

$y = 0\ when\ x = 1.$

Q26. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, prove that $(\vec{a}-\vec{d})$ is parallel to $(\vec{b}-\vec{c})$,

where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

Q27. Find the shortest distance between the lines whose vector equation are

$\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k}).$

Q28. If P(A) = $\frac{2}{5}, P(B) = \frac{1}{3}, P(A\cap B) = \frac{1}{5}, then\ find\ P(\bar{A}/B).$

SANSKRITI THE CIVIL SERVICES SCHOOL

OR

An urn contains 5 red and 2 black balls. Two balls are randomly drawn, without replacement. Find the probability distribution of the number of black balls drawn.

Section-IV

Q29. Show that the function f on A= $R-\left\{\frac{2}{3}\right\}$ defined as $f(x)=\frac{4x+3}{6x-4}$ is one-one and onto.

Q30. Find the values of a and b so that the function $f(x) = \begin{cases} x^2+3x+a, & x \le 1 \\ bx+2, & x > 1 \end{cases}$ is differentiable for $x \in R$.

OR

If $x = a\sin pt$ and $y = b\cos pt$, then find $\frac{d^2y}{dx^2}$ at $t = 0$.

Q31. If $y = x^{sinx} + sin^{-1}\sqrt{x}$, find $\frac{dy}{dx}$.

Q32. Find the intervals in which the function $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$ is increasing or decreasing.

Q33. Find $\int \frac{sinx}{(1-cosx)(2-cosx)} dx$

OR

Find $\int x\log(1+x)\,dx$

Q34. Using integration, find the area of the region bounded by the curve $y = \sqrt{1-x^2}$, the line $y = x$ and the positive x- axis.

OR

Using integration, find the value of "a" if area between $x = y^2$ and $x = 4$ is divided into equal parts by the line $x = a$.

Q35. Solve the following differential equation: $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$.

OR

Solve the differential equation: $(1+x^2)dy + 2xydx = cotxdx,\ x \neq 0$

Section-V

Q36. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$, $find\ A^{-1}$ and hence use A^{-1} to solve the system of equations :

$$2x + y - 3z = 13, \qquad 3x + 2y + z = 4, \qquad x + 2y - z = 8.$$

OR

If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, $find\ (AB)^{-1}$.

Q37. Find the distance of the point (-2,3, -4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane 4x + 12y – 3z + 1 = 0.

OR

Find the equation of the plane through the line of intersection of $\vec{r}.\left(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right) = 1$ and $\vec{r}.(\hat{\imath} - \hat{\jmath}) + 4 = 0$ and perpendicular to the plane $\vec{r}.\left(2\hat{\imath} - \hat{\jmath} + \hat{k}\right) + 8 = 0$. Hence, find whether the plane thus obtained contains the line x - 1 = 2y - 4 = 3z - 12.

Q38. Solve the following linear programming problem (L.P.P.) graphically.

Maximize $Z = 3x + 9y$ subject to the constraints

$x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0$

$y \geq 0$

Answers

Assignment 1

1. $[0,1[$

3. $g \circ f(x) = 2x$

$f \circ g(x) = 8x$

4. x

5. 1

6. 27. 0

8. $f^{-1}(x) = \frac{x}{1-x}$

9. $[(2,5)] = \{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9)\}$

12. $fog(x) = \begin{cases} 0, x \geq 0 \\ -4x, x < 0 \end{cases}$

$gof(x) = 0,\ for$ all x

$fog(-3) = 12$

$fog(5) = 0, gof(-2) = 0$

13. $f^{-1}(x) = (x-27)^{\frac{1}{3}}$

$f^{-1}(0) = -3$

14.(i) $\{0,2,4\}$

(ii) R is not an equivalence relation

16. $\left(\frac{x-7}{2}\right)^{\frac{1}{3}}, 1$

Assignment 2:

1. $\frac{\pi}{2}$

2. $\frac{-\pi}{10}$

3. $\pi - \sec^{-1} x$

4. $\frac{1}{2}$

5. $\frac{1}{\sqrt{3}}$

6. $\frac{\sqrt{3}}{2}$

8. 0, -1

9. -1

11.a)

$\frac{1}{2}\tan^{-1} x$

b)

$\frac{x}{2}$ / $\frac{\pi}{2} - \frac{x}{2}$

c)

$\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x$

SANSKRITI THE CIVIL SERVICES SCHOOL

14. $x=\frac{1}{4}$

15. $0,\frac{1}{2}$

Assignment No. 3

1. $x=3; y=3$

2. I

3. $\begin{bmatrix} 2 & -1 & -2 \\ 3 & 4 & -1 \end{bmatrix}$

4. 0

5. $\begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$

6. $\begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$

7. $\begin{bmatrix} 3 & 3/2 & 5/2 \\ 3/2 & 4 & 4 \\ 5/2 & 4 & 8 \end{bmatrix} + \begin{bmatrix} 0 & 1/2 & 9/2 \\ -1/2 & 0 & -1 \\ -9/2 & 1 & 0 \end{bmatrix}$

8. $x=-1; x=-2$

10. $\begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$

11. x = 200, y=1000

12. (i) $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$

Assignment No. 4

1. 36

2. 25

3. 0

4. 64

5. -15

6. 216

9. a) $x=2, y=3, z=5$

9.b) $x=1, y=2, z=5$

10. $x=2, y=-1, z=4$

11. $x=1, y=1, z=1$

12. $\begin{bmatrix} -7 & 3 \\ 12 & -5 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

14. $k=-6$ or 14

SANSKRITI THE CIVIL SERVICES SCHOOL

Assignment No. 5

1. $\frac{1}{2}$ 2. $\frac{2(6^x \log 6)}{1+6^{2x}}$ 4. $\frac{2}{1+x^2}$ 5. Discontinuous at $x=-2,-1,0,1,2$ 6. $\frac{1}{2(1+x^2)}$

7. $\frac{-1}{(1+x)2\sqrt{x}}$ 8. $\frac{1+x^2}{2}$

9. 5 11. $\frac{1}{2\sqrt{12}}$ 12. $a=-\frac{3}{2}, c=\frac{1}{2}$ & $b \in R - \{0\}$

13. Not derivable

14. $a=2,\ b=-1$ 15. Continuous

16. $x=\pi$ 17. $x^{\sin x}\left[\frac{\sin x}{x}+\cos x \log x\right]+\frac{1}{\sqrt{1-x}}\ \frac{1}{2\sqrt{x}}$ 22. $\frac{2\sqrt{2}}{a}$ 23. $\frac{32}{27a}$ 24. $\frac{-1}{2}$

Assignment No. 6

1. $\frac{\sqrt{2}}{4\pi} cm/\sec$ 2. $-\frac{2}{75} radian/\sec$ 3. 5.02, 3.074, 0.1925 4. $(-1,\infty)$ function is increasing.

5. $(-\infty,-1)$ & $(-1,1)$ function decreasing, $(1,3)$ & $(3,\infty)$ function increases; point of minima is 1, points of inflexion are $-1,3$. 6. $x=3$ point of maxima, $x=0,5$ are points of minima. Minimum values: $f(0)=105$; $f(5)=\frac{545}{4}$; Maximum value: $f(3)=\frac{609}{4}$.

7. $(4,-4)$ 8. $y+3x=3$ & $y=7x-14$ 9. Increasing in the interval $(-\infty,1)$ & $(2,\infty)$ and decreasing in the interval $(1,2)$ 10. $\left(0,\frac{\pi}{4}\right)$ decreasing and $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ increasing 11.a) $x=\frac{\pi}{6}$ is a point of maxima and the maximum value $\sqrt{3}+\frac{\pi}{6}$. Point of minima is $x=\frac{5\pi}{6}$ and the minimum value is $-\sqrt{3}+\frac{5\pi}{6}$. 11(b) $x=\frac{\pi}{3}$ is a point of maxima and the maximum value is $\sqrt{3}-\frac{\pi}{3}$; $x=-\frac{\pi}{3}$ is a point of minima and the minimum value is $-\sqrt{3}+\frac{\pi}{3}$

12. $y-\frac{\pi}{4}+\frac{1}{\sqrt{2}}=(\sqrt{2}-1)\left(x-\frac{\sqrt{2}-1}{\sqrt{2}}\right)$ & $y-\frac{\pi}{4}+\frac{1}{\sqrt{2}}=\frac{-1}{(\sqrt{2}-1)}\left(x-\frac{\sqrt{2}-1}{\sqrt{2}}\right)$

15. (-2,-8) 16. $x+2^{\frac{2}{3}}y=2+2^{\frac{2}{3}}$

Answers of Indefinite Integral questions

1. $\log\left(1+x^2\right)+c$ 2. $-\frac{1}{8}\log\left|9-4x^2\right|+c$ 3. $\log\left|e^x+e^{-x}\right|+c$ 4. $\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+c$

5. $-\log\left|1+\cos x\right|+c$ 6. $\frac{1}{2}\log\left|2\sin x+3\cos x\right|+c$ 7. $-\frac{1}{4}\tan\left(7-4x\right)+c$ 8. $\frac{1}{2}\tan\left(2x-3\right)+c$

9. $\frac{2}{3a}\left(ax+b\right)^{\frac{3}{2}}+c$ 10. $\frac{e^{2x+3}}{2}+c$ 11. $\frac{1}{2}\log\left|\sec 2x+\tan 2x\right|+c$ 12. $-\frac{1}{3}\cos\left(3x-1\right)+c$

13. $\frac{1}{3}\left(\log\left|x\right|\right)^3+c$ 14. $\log\left|1+\log x\right|+c$ 15. $\frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}}+c$ 16. $\frac{1}{7}\left(x^3-1\right)^{\frac{7}{3}}+\frac{1}{4}\left(x^3-1\right)^{\frac{4}{3}}+c$

17. $-\frac{1}{2e^x}+c$ 18. $-\frac{1}{4}\cos\left(\tan^{-1}x^4\right)+c$ 19. $\frac{9}{82}\log\left|5\sin x+4\cos x\right|+\frac{22}{41}x+c$

20. $\frac{x}{2}-\frac{1}{2}\log\left|\cos x+\sin x\right|+c$ 21. $\frac{x}{2}-\frac{1}{2}\log\left|\cos x-\sin x\right|+c$ 22. $\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+c$

23. $\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)+c$ 24. $-\frac{3}{4}\cos x+\frac{\cos 3x}{12}+c$ 25. $\frac{1}{4}\left(\frac{\sin 3x}{3}+3\sin x\right)+c$

26. $\frac{x}{2}-\frac{1}{8}\sin\left(4x+10\right)+c$ 27. $-\frac{1}{2}\cos\left(2x+1\right)+\frac{1}{6}\cos^3\left(2x+1\right)+c$ 28. $-\frac{1}{14}\cos 7x+\frac{1}{2}\cos x+c$

29. $\frac{1}{2}\left(\frac{1}{4}\sin 4x-\frac{1}{12}\sin 12x\right)+c$ 30. $\frac{1}{4}\left(\frac{1}{12}\sin 12x+x+\frac{1}{8}\sin 8x+\frac{1}{4}\sin 4x\right)+c$

31. $\frac{3x}{8}-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+c$ 32. $\frac{3x}{8}+\frac{1}{8}\sin 4x+\frac{1}{64}\sin 8x+c$ 33. $\frac{1}{3}\tan^3 x-\tan x+x+c$

34. $\log\left|\tan x\right|+\frac{1}{2}\tan^2 x+c$ 35. $\frac{1}{8}\left(x-\frac{\sin 4x}{4}\right)+c$ 36. $\frac{1}{16}\left(-\frac{3}{4}\cos 2x+\frac{1}{12}\cos 6x\right)+c$

37. $\frac{1}{\sin\left(a-b\right)}\log\left|\frac{\cos\left(x-a\right)}{\cos\left(x-b\right)}\right|+c$ 38. $\frac{1}{\cos\left(a-b\right)}\log\left|\frac{\sin\left(x+a\right)}{\cos\left(x+b\right)}\right|+c$ 39. $-\frac{\cos^3 x}{3}+\frac{\cos^5 x}{5}+c$

40. $\frac{1}{128}\left(3x-\sin 4x+\frac{1}{8}\sin 8x\right)+c$ 41. $-\left(\cos x+\frac{1}{5}\cos^5 x-\frac{2}{3}\cos^3 x\right)+c$ 42. $\tan^{-1}x^3+c$

$43.\ \frac{1}{2}\log\left|2x+\sqrt{1+4x^2}\right|+c$ $44.\ \log\left|\frac{1}{2-x+\sqrt{x^2-4x+5}}\right|+c$ $45.\ \frac{1}{5}\sin^{-1}\frac{5x}{3}+c$

$46.\ \log\left|\tan x+\sqrt{\tan^2 x+4}\right|+c$ $47.\ \log\left|\sin x-1+\sqrt{\sin^2 x-2\sin x-3}\right|+c$

$48.\ \frac{1}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|+c$ $49.\ \frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right)+c$

$50.\ \frac{5}{6}\log\left|3x^2+2x+1\right|-\frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+c$ $51.\ \frac{1}{2}\log\left|x^2-2x-5\right|+\frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|+c$

$52.\ \log\left|x+1+\sqrt{x^2+2x+2}\right|+c$ $53.\ \sin^{-1}\left(\frac{x+3}{2}\right)+c$

$54.\ 6\sqrt{x^2-9x+20}+34\log\left|x-\frac{9}{2}+\sqrt{x^2-9x+20}\right|+c$ $55.\ -\sqrt{4x-x^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)+c$

$56.\ 2\log\left|\sin^2\phi-4\sin\phi+5\right|+7\tan^{-1}(\sin\phi-2)+c$ $57.\ -\frac{1}{\cos x+\sin x}+c$

$58.\ \sin^{-1}(\sin x-\cos x)+c$ $59.\ \frac{1}{40}\log\left|\frac{5+(\sin x-\cos x)}{5-(\sin x-\cos x)}\right|+c$ $60.\ 2\log\left|\sqrt{x}-1\right|+c$

$61.\ 2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+c$ $62.\ \log\frac{(x+2)^2}{|x+1|}+c$ $63.\ \frac{1}{6}\log\left|\frac{x-3}{x+3}\right|+c$

$64.\ \log|x-1|-5\log|x-2|4\log|x-3|+c$ $65.\ \frac{x}{2}+\log|x|-\frac{3}{4}\log|1-2x|+c$

$66.\ \frac{5}{2}\log|x+1|-\frac{1}{10}\log|x-1|-\frac{12}{5}\log|2x+3|+c$ $67.\ \frac{x^2}{2}+\frac{1}{2}\log|x+1|+\frac{3}{2}\log|x-1|+c$

$68.\ \frac{2}{9}\log\left|\frac{x-1}{x+2}\right|-\frac{1}{3(x-1)}+c$ $69.\ 3\log|x-2|+\frac{7}{x+2}+c$

$70.\ -5\log|x-1|+\frac{5}{x-1}+\frac{1}{(x-1)^2}+5\log|x-2|+c$ $71.\ -\log|x-1|+\frac{1}{2}\log\left(1+x^2\right)+\tan^{-1}x+c$

72. $\frac{1}{2}\log|x-1|-\frac{1}{4}\log\left(1+x^2\right)+\frac{1}{2}\tan^{-1}x+c$ 73. $\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+c$

74. $-\frac{1}{4x}+\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)+c$ 75. $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+c$ 76. $\frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right|+c$

77. $\frac{1}{3\sqrt{2}}\left\{\tan^{-1}\frac{x}{\sqrt{2}}+\tan^{-1}\left(\sqrt{2}x\right)\right\}+c$ 78. $x+\frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}-3\tan^{-1}\frac{x}{2}+c$ 79. $\frac{1}{2}\log\left(\frac{x^2+1}{x^2+3}\right)+c$

80. $\log\left|\frac{2-\sin x}{1-\sin x}\right|+c$ 81. $-\frac{1}{8}\log\left|\frac{1+\sin x}{1-\sin x}\right|+\frac{1}{4\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right|+c$

82. $-\frac{1}{3}\log|1+\tan\theta|+\frac{1}{6}\log\left|\tan^2\theta-\tan\theta+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan\theta-1}{\sqrt{3}}\right)+c$

83. $\frac{1}{6}\log|1-\cos x|+\frac{1}{2}\log|1+\cos x|-\frac{2}{3}\log|1+2\cos x|+c$ 84. $-x\cos x+\sin x+c$

85. $e^x\left(x^2-2x+2\right)+c$ 86. $\frac{x^3}{3}\log x-\frac{x^3}{9}+c$ 87. $\frac{1}{4}\left(2x^2-1\right)\sin^{-1}x+\frac{x\sqrt{1-x^2}}{4}+c$

88. $\frac{x^2}{2}\tan^{-1}x-\frac{x}{2}+\frac{1}{2}\tan^{-1}x+c$ 89. $x\sin^{-1}x+\sqrt{1-x^2}+c$

90. $\left(\sin^{-1}x\right)^2x+2\sqrt{1-x^2}\sin^{-1}x-2x+c$ 91. $x\log x-x+c$ 92. $x(\log x)^2-2(x\log x-x)+c$

93. $\frac{x^2}{2}(\log x)^2-\frac{x^2}{2}\log x+\frac{x^2}{4}+c$ 94. $x\tan^{-1}x-\frac{1}{2}\log\left(1+x^2\right)+c$

95. $\frac{1}{2}\left(\sec x\tan x+\log|\sec x+\tan x|\right)+c$ 96. $\frac{1}{2}\left(-\operatorname{cosec}x\cot x+\log|\operatorname{cosec}x-\cot x|\right)+c$

97. $e^x\sin x+c$ 98. $\frac{e^x}{1+x}+c$ 99. $e^x\tan\frac{x}{2}+c$ 100. $e^x\frac{1}{x}+c$

101. $\frac{e^x}{(x-1)^2}+c$ 102. $e^x-\frac{2e^x}{x+1}+c$ 103. $\frac{x}{\log x}+c$ 104. $x\log(\log x)-\frac{x}{\log x}+c$

105. $e^x\log x-\frac{e^x}{x}+c$ 106. $\frac{e^{2x}}{5}(2\sin x-\cos x)+c$ 107. $\frac{e^{-x}}{2}(\sin x-\cos x)+c$

108. $\frac{e^x}{2}-\frac{e^x}{10}(\cos 2x+2\sin 2x)+c$ 109. $\frac{e^{2x}}{4}+\frac{e^{2x}}{8}(\cos 2x+2\sin 2x)+c$

110. $\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+c$ 111. $\frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}(2x)+c$

112. $\frac{1}{2}\left\{x\sqrt{(x+2)^2-9}-9\log\left|x+2+\sqrt{(x+2)^2-9}\right|\right\}+c$

113. $\frac{1}{2}\left\{(x+2)\sqrt{x^2+4x+6}+2\log\left|x+2+\sqrt{x^2+4x+6}\right|\right\}+c$

114. $\frac{2x-3}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+c$

115. $-\frac{1}{3}(3-4x-x^2)^{\frac{3}{2}}+\frac{1}{2}(x+2)\sqrt{3-4x-x^2}+\frac{7}{2}\sin^{-1}\frac{x+2}{\sqrt{7}}+c$

116. $\frac{1}{\sqrt{3}}\log\left|\frac{\tan\frac{x}{2}-2-\sqrt{3}}{\tan\frac{x}{2}-2+\sqrt{3}}\right|+c$ 117. $\frac{2}{3}\tan^{-1}\left(3\tan\frac{x}{2}\right)+c$ 118. $\frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right|+c$

119. $\sqrt{2}\tan^{-1}\left(\frac{\tan\frac{x}{2}+1}{\sqrt{2}}\right)+c$ 120. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$ 121. $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+c$

122. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right)+\frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+c$

123. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right)-\frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+c$

124. $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)+c$ 125. $\frac{1}{3}\tan^{-1}\left(\frac{x^2-1}{3x}\right)+c$

126. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+\frac{1}{2\sqrt{2}}\log\left|\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right|+c$

127. $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\cot x-1}{\sqrt{2\cot x}}\right)-\frac{1}{2\sqrt{2}}\log\left|\frac{\cot x-\sqrt{2\cot x}+1}{\cot x+\sqrt{2\cot x}+1}\right|+c$

128. $\frac{1}{\sqrt{15}}\tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{5}}\right)+c$ 129. $\frac{1}{2}\tan^{-1}(2\tan x)+c$ 130. $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)+c$

131. $\frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right|+c$ 132. $\tan^{-1}(\tan^2 x)+c$ 133. $\frac{1}{5}\log\left|\frac{\tan x-2}{2\tan x+1}\right|+c$

134. $\frac{1}{2\sqrt{5}}\log\left|\frac{\sqrt{5}\tan x-1}{\sqrt{5}\tan x+1}\right|+c$ 135. $-\frac{1}{2(2\tan x+3)}+c$ 136. $2\sqrt{\tan x}+c$

137. $a\sin^{-1}\frac{x}{a}+\sqrt{a^2-x^2}+c$ 138. $-\cos\alpha\sin^{-1}\left(\frac{\cos x}{\cos\alpha}\right)-\sin\alpha\log\left|\sin x+\sqrt{\sin^2 x-\sin^2\alpha}\right|$

139. $-\log\left|\cos x+\frac{1}{2}+\sqrt{\cos^2 x+\cos x}\right|+c$

140. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+(\sin x-\cos x)}{\sqrt{3}-(\sin x-\cos x)}\right|+\tan^{-1}(\sin x+\cos x)+c$

141. $-\frac{1}{4}\sin^{-1}\left(\frac{\cos^2 2x}{3}\right)+c$ 142. $2\tan^{-1}\sqrt{x+\frac{1}{x}+1}+c$ 143. $\frac{4}{15}\left(1-\frac{1}{x^3}\right)^{\frac{5}{4}}+c$

144. $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+c$ 145. $\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$

<u>Assignment No. 7 (a)</u>

1. $\sqrt{x^2+1}-2\log\left|x+\sqrt{x^2+1}\right|+c$ 2. $\frac{2}{3}x^{\frac{3}{2}}+\frac{x^3}{3}+c$ 3. $\frac{1}{2}\log\left|\operatorname{cosec}(x+\frac{\pi}{6})-\cot(x+\frac{\pi}{6})\right|$

4. $\log|x+\log\cos x|$ 5. 8 6. $x\sin(\log x)$ 7. $k=\frac{1}{\log 2}$ 8. $\frac{1}{24}\log\frac{3x-4}{3x+4}$

9. $-\frac{1}{2}e^{-x^2}+c$ 10. $\tan^{-1}e^x+c$ 11. $\frac{1}{3}\log\frac{2\tan x+1}{\tan x-2}+C$ 12. $-\sqrt{1+2\cot x}+c$

13. $\cot x+\frac{1}{7}\cot^7 x+\frac{3}{5}\cot^5 x+\cot^3 x+c$

14. $\frac{1}{(\log 5)^3}5^{5^{5^x}}+c$

15. $\frac{1}{3}\log\left|x^3+\sqrt{x^6-a^6}\right|+c$

16. $\frac{x^2}{2}+2x+\frac{3}{2}\log\left|x^2-x+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\frac{2x-1}{\sqrt{3}}+c$

17. $\sqrt{-x^2+5x+6}+\frac{1}{2}\sin^{-1}(2x-5)+c$

18. $\log\left|\frac{2e^x+1}{2e^x+2}\right|+c$

19. $-3\sqrt{6+x-2x^2}+\frac{13}{2\sqrt{2}}\sin^{-1}\frac{4x-1}{7}+c$

20. $\sin^{-1}(2\sin x-1)+c$

21. $\frac{-1}{3(3\tan x+1)}+C$

22. $\log(\log(\log x))+C$

23. $\sqrt{2}(\sin^{-1}(\sin\theta-\cos\theta))+C$

24. $\frac{1}{2}\log\left|\frac{\tan x}{\tan x+2}\right|+C$

25. $(\log x)^2+C$

26. $\log\left|\cos x+\frac{1}{2}+\sqrt{\cos^2 x+\cos x}\right|+C$

27. $\frac{-2}{\sqrt{23}}\tan^{-1}\frac{2\tan\frac{x}{2}+1}{\sqrt{23}}+c$

28. $\frac{1}{3\sqrt{2}}\tan^{-1}\frac{x^2-9}{3\sqrt{2}x}+c$

29. $\frac{1}{2a^3b^3}\left\{(a^2+b^2)\tan^{-1}(\frac{a\tan x}{b})+\frac{(a^2-b^2)ab\tan x}{a^2\tan^2 x+b^2}\right\}+c$

Assignment 7 (b)

1. $\frac{1}{4}\log|x-1|+\frac{3}{4}\log|x+1|+\frac{1}{2(x+1)}+c$

2. $3\left[x\cos^{-1}x-\sqrt{1-x^2}\right]+c$

3. $\frac{e^x}{(x+1)^2}+c$

4. $-\frac{1}{3}\log|\tan x+1|+\frac{1}{6}\log\left|\tan^2 x-\tan x+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\frac{2\tan x-1}{\sqrt{3}}+c$

5. $e^{\frac{-x}{2}}\sec\frac{x}{2}+c$

6. $-\frac{\tan^{-1}x}{\sqrt{1+x^2}}+\frac{x}{\sqrt{1+x^2}}+c$

7. $-\frac{\log(x+2)}{x+2}-\frac{1}{x+2}+c$

8. $-\frac{1}{2}\log|1-\cos x|+\frac{1}{18}\log|1+\cos x|+\frac{4}{9}\log|5-4\cos x|+c$

9. $2\left[\sqrt{x}e^{\sqrt{x}}-e^{\sqrt{x}}\right]+c$

10. $\frac{3}{2}\sqrt{x}\sin\sqrt{x}+\frac{3}{2}\cos\sqrt{x}+\frac{1}{6}\sqrt{x}\sin 3\sqrt{x}+\frac{1}{18}\cos 3\sqrt{x}+c$

11. $\frac{9}{13}\left[-\frac{1}{3}e^{-3x}\cos 2x+\frac{2}{9}e^{-3x}\sin 2x\right]+c$

12. $e^{x}\left[-\log\cos x\right]+c$

13. $\frac{1}{2}e^{2x}\cot 2x+c$

14. $x-\log x+\frac{1}{2}\log\left|x^2+1\right|-\tan^{-1}x+c$

15. $-\frac{1}{3}\left(3-x-x^2\right)^{\frac{3}{2}}+\frac{1}{2}\left[\frac{2x+1}{4}\sqrt{3-x-x^2}+\frac{13}{8}\sin^{-1}\frac{2x+1}{\sqrt{13}}\right]+c$

16. $\frac{x^3}{3}\operatorname{cosec}^{-1}x+\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2-1}-\frac{1}{2}\log\left|x+\sqrt{x^2-1}\right|\right]+\frac{1}{3}\log\left|x+\sqrt{x^2-1}\right|+c$

17. $x\log(\log x)-\frac{x}{\log x}+c$

18. $\left(x^2+2x-3\right)^{\frac{3}{2}}-2\left[\frac{x+1}{2}\sqrt{x^2+2x-3}-2\log\left|x+1+\sqrt{x^2+2x-3}\right|\right]+c$

Assignment 8

1. 0.5 2. -2 3. 2

4. 2 5. $e-1$

6. $\frac{1}{2}$ 7. $\frac{\pi}{4}$

8. $a=-1, b=1$ 9. (a) $1-\log 2$ (b) $\sqrt{2}\pi$

(c) $\frac{\pi}{\sqrt{35}}$ (d) $\frac{\pi}{12}$ (e) 0 (f) $\frac{\pi^2}{16}$ (g) $\frac{\pi}{2}-\log 2$

(h) $\frac{\pi}{8}\log 2$ (i) 2 (j) $\frac{\pi}{2}$ (k) $\frac{63}{2}$ (l) π^2 (m) $\frac{\pi^2}{6\sqrt{3}}$

10. (i) $\frac{-7}{6}$ (ii) $\frac{e^2\left(e^4-1\right)}{2}$ (iii) 32

(iv) e^4+7

SANSKRITI THE CIVIL SERVICES SCHOOL

Assignment 9

1. $\left(\frac{16}{3}-\frac{4}{3}\sqrt{2}\right)$ sq units 2. $\frac{4\sqrt{3}+16\pi}{3}$ sq units

3. $\frac{16a^2}{3}$ sq units 4. 7 sq units 5. 6 sq units

6. $\frac{4}{3}$ sq units 7. (π) sq units

8. 10 sq units 9. $\frac{62}{3}$ sq units 10. $\frac{64}{3}$ sq units

11. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ sq units 12. $\left(\frac{5\pi}{2}-\sin^{-1}\frac{1}{4}-4\sin^{-1}\frac{7}{8}-\frac{\sqrt{15}}{2}\right)$ sq units

13. 4 sq units 14. 18 sq units 15. $\left(\frac{5}{2}(\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{1}{\sqrt{5}})-\frac{1}{2}\right)$ sq units

Assignment10

1. $\frac{d^2y}{dx^2}-y=0$ 2. $y=x$ 5. $(i)\,2,4\ (ii)\,2, n.d.$
$(iii)\,1,2\ (iv)\,2,3$

6. $(i)\,e^{\tan^{-1}x}\ (ii)\,\frac{1}{x}\ (iii)\,\frac{1}{y^2}\ (iv)\,e^{2x}$ 7. $(i)\log|y|=2x+2\log|x-1|-4$

$(ii)\log|x|+x=-\log|y|-y+c\ (iii)\,e^{2x}y=-\cos x+1$

$(iv)\,x=2y^2\ (v)\log\left|1+\tan\frac{x+y}{2}\right|=x+c$

$(vi)\,c-\sqrt{1+y^2}=\sqrt{1+x^2}+\frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right|$

$(vii)\,x=y+1-\log|x+y+2|=x+c$

$(viii)\,x^2\sin x+2x\cos x-2\sin x-x^2y+c=0$

$(ix)-\tan^{-1}y=\tan^{-1}e^x-\frac{\pi}{2}$

$(x)\log|1+y|=x-\frac{x^2}{2}+c$

Answers to Vectors

1. $0 \le |k\vec{a}| \le 6$
2. $a = -4$
3. $\frac{11}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$
4. $5\hat{i} + 5\hat{k}$
5. $\left\langle \frac{2}{7}, \frac{3}{7}, -\frac{6}{7} \right\rangle$, *Scalar* components : 4,6, -12 and vector components : $4\hat{i}, 6\hat{j}, -12\hat{k}$
6. $\pm\left(\frac{6}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k}\right)$
7. $|\overrightarrow{AB}| = \sqrt{14}, |\overrightarrow{BC}| = \sqrt{21}, |\overrightarrow{CA}| = \sqrt{35}$
8. $\left(3\vec{a} + 5\vec{b}\right)$
9. $\frac{\sqrt{34}}{2}$
10. 8
11. 5
12. $120°$
13. Prove dot product to be zero
14. $\pm 2\sqrt{10}$
15. 5
16. $\sqrt{21}$
17. $\lambda = 1,\ \frac{1}{7}\left(3\hat{i} + 6\hat{j} - 2\hat{k}\right)$
18. $45°, 45°$
19. Prove
20. $60°$
21. $\pm 10\left(\hat{i} - \hat{j} + \hat{k}\right)$
22. $\pm \frac{1}{\sqrt{165}}\left(-10\hat{i} - 7\hat{j} + 4\hat{k}\right)$
23. $\frac{\pi}{2}$, $2\hat{i} - 26\hat{j} - 10\hat{k}$
24. ± 12
25. Prove
26. $\left(\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$

SANSKRITI THE CIVIL SERVICES SCHOOL

Assignment11(a)

1. $\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j}$ 2. $\sqrt{a^2+b^2}$ 3. $\hat{i}+\hat{j},-\hat{i}-\hat{j}$

4. 5 5. $\frac{3}{7},\frac{-6}{7},\frac{2}{7}$

6. ± 6 7. $\frac{\pi}{3},\frac{2\pi}{3}$ 8. $\pm 2\sqrt{3}$ 9. $\frac{9\hat{i}+11\hat{j}+15\hat{k}}{3}$

10. (i) $\cos^{-1}\frac{1}{\sqrt{3}},\cos^{-1}\frac{1}{\sqrt{3}},\cos^{-1}\frac{-1}{\sqrt{3}}$

(ii) $\frac{3\pi}{4},\frac{3\pi}{4},\frac{\pi}{2}$ 12. $\frac{-2\hat{i}+\hat{j}+5\hat{k}}{\sqrt{30}}$ 13. $2\hat{i}-2\hat{j}+\hat{k}$

14. $\sqrt{114}$ 15. $\frac{5}{\sqrt{6}},\frac{5}{6}\left(\hat{i}-2\hat{j}+\hat{k}\right)$

17. -3 18. $x=-6, y=2$

19. $4\hat{i}+\hat{j}$ 20. ± 6 21. 90° 22. 1

Assignment11(b)

1. $2,-1$ 2. $\frac{\pi}{4}$ 3. $3\sqrt{2}$

4. $\frac{-\hat{i}+2\hat{j}+\hat{k}}{\sqrt{6}},\frac{\hat{i}-\hat{k}}{\sqrt{2}}$ 5. $5\sqrt{17}$ *sq units*

7. $3\frac{27}{2}$ 8. $\hat{i}+3\hat{j}+3\hat{k}$

10. 264 *cubicunits*

12. $\lambda=1$ 13. $\alpha=-2,-3$

14. -1

19. $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j},\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}$

20. $\pm\frac{100}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$

विद्यया च विमुच्यते SANSKRITI THE CIVIL SERVICES SCHOOL

Answers to Three-Dimensional Geometry

1. $60°$ or $120°$
2. $\left\langle \frac{3}{\sqrt{77}}, -\frac{2}{\sqrt{77}}, \frac{8}{\sqrt{77}} \right\rangle$
3. $p = 18, q = \frac{2}{3}$
4. $(-2,1,7)$ and $(-3,-6,10)$
5. $\left\langle \frac{6}{7}, \frac{2}{7}, -\frac{6}{7} \right\rangle$
6. $\langle 3,2,-1 \rangle$, $\vec{r} = 7\hat{i} - 5\hat{j} + \lambda(3\hat{i} + 2\hat{j} - \hat{k})$
7. $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$, $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$

   $\vec{r} = -\hat{i} - 2\hat{j} - \hat{k} + \mu(3\hat{i} + 5\hat{j} + 3\hat{k})$, λ and μ are parameters , $\frac{x+1}{3} = \frac{y+2}{5} = \frac{z+1}{3}$

   $D(4,7,6)$
8. $(-2,-1,3)$ and $(4,3,7)$
9. $(4,0,-1)$
10. $90°$
11. $p = 7$, $\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$
12. $(1,0,7)$
13. $\sqrt{10}$
14. $\vec{r} = \hat{i} - \hat{j} + \hat{k} + \lambda(10\hat{i} - 4\hat{j} - 7\hat{k})$, $\frac{x-1}{10} = \frac{y+1}{-4} = \frac{z-1}{-7}$
15. $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ and $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$
16. $2\sqrt{29}$
17. $\frac{5}{\sqrt{29}}$
18. $\frac{x-1}{3} = \frac{y-1}{10} = \frac{z-1}{17}$
19. $\vec{r}.(2\hat{i} - 3\hat{j} + 6\hat{k}) = 35$
20. $\vec{r}.(\hat{i} - 3\hat{j} + 2\hat{k}) + 3 = 0$, $x - 3y + 2z + 3 = 0$
23. 13
24. $(2,2,0)$
25. 1 unit.

SANSKRITI THE CIVIL SERVICES SCHOOL

26. $\frac{17}{2}$

27. $\sqrt{6}$, $\left(0,\frac{5}{2},0\right)$

28. $3\hat{i}+\frac{7}{2}\hat{j}+\frac{11}{2}\hat{k}$, $\frac{\sqrt{14}}{2}$, $(4,4,7)$

29. $6x-3y+z-3=0$

30. $\vec{r}.(\ 7\hat{j}+5\hat{k}\ )=35$

31. $\vec{r}.(\ 8\hat{i}+5\hat{j}+4\hat{k}\ )-12=0$

32. $x+y+z=0$

33. $8x+y-5z-7=0$, It contains the given line.

34. $k=\frac{9}{2}$, $5x-2y-z-6=0$

35. $9x+2y-7z-21=0$

36. $(1,-2,7),2:1$

37. $4x-3y+2z=12, \frac{12}{\sqrt{29}}$ *units*

38. $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$

39. $6x+3y-2z=18$ and $2x-3y-6z=6$

40. $5x+2y+5z-9=0$

41. $\cos^{-1}\left(\frac{15}{\sqrt{731}}\right)$

42. $0°$

43. $\left(\frac{17}{3},0,\frac{23}{3}\right)$, $\sin^{-1}\left(\frac{3}{\sqrt{38}}\right)$

44. $\langle 1,1,\pm\sqrt{2}\rangle$, $x+y+\sqrt{2}z-1=0$ and $x+y-\sqrt{2}z-1=0$

45. $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(\ -3\hat{i}+5\hat{j}+4\hat{k}\ )$

46. $\vec{r}.(\ 5\hat{i}+7\hat{j}+\hat{k}\ )=1$, $(0,-1,8)$

47. $\vec{r}.(\ 33\hat{i}+45\hat{j}+50\hat{k}\ )=41$

48. $7x+9y-10z-27=0$

49. $\frac{13}{7}$

50. $2x+y+2z-3=0$ & $x-2y+2z+3=0$ $\vec{r}.(\ 2\hat{i}+\hat{j}+2\hat{k}\ )-3=0$

$\vec{r}.(\ \hat{i}-2\hat{j}+2\hat{k}\ )+3=0$.

51. 1 unit

52. $4x-6y+12z+7=0$

Assignment12

1.$\frac{9}{\sqrt{50}}$ 2.$\vec{r}.\hat{j}=3$ 3.$3x+2y-z-3=0$

4.$-\frac{10}{7}$ 5.$\vec{r}=2\hat{i}-3\hat{j}+4\hat{k}+\mu\left(2\hat{i}+5\hat{k}\right)$

6.$\frac{9\hat{i}+11\hat{j}+15\hat{k}}{3}$ 7.$\left(-\frac{1}{3},\frac{1}{3},1\right),\vec{r}=\frac{-1}{3}\hat{i}+\frac{1}{3}\hat{j}+\hat{k}+\lambda\left(\frac{1}{3}\hat{i}+\frac{1}{6}\hat{j}-\hat{k}\right)$

8.$acute\ angle=\sin^{-1}\frac{1}{3\sqrt{2}}$ 9.$\frac{7}{3},\frac{7}{2},7$

10.$z=5$ 11.$(3,5,9),-18x+22y-5z=11$ 12.$\vec{r}.\left(7\hat{i}+9\hat{j}-10\hat{k}\right)=27$

13.$\vec{r}.\left(4\hat{i}-\hat{j}-2\hat{k}\right)=6$ 14.$(a)\sqrt{62}units\ (b)\frac{84}{29\sqrt{2}}units$

15.$\vec{r}.\left(18\hat{i}+17\hat{j}+4\hat{k}\right)=49$ 17.$\vec{r}.\left(\hat{i}+3\hat{j}-4\hat{k}\right)=-6,\vec{r}.\left(-2\hat{i}+4\hat{j}+4\hat{k}\right)=-6$

18.$5y-5z-6=0,\frac{6}{5\sqrt{2}}units$ 21.$6units$ 22.$\frac{10}{3\sqrt{3}}units$

23.$\left(\frac{17}{3},0,\frac{23}{3}\right)$ 24.$x-2y+2z+2=0,x-2y+2z-4=0$ 25. $\frac{x-1}{3}=\frac{y-1}{10}=\frac{z-1}{17}$

26. $\vec{r}.\left(5\hat{i}+7\hat{j}+\hat{k}\right)=1,-\hat{j}+8\hat{k}$

Assignment13

1.$\frac{3}{8}$ 2.0.9 3.$\frac{1}{2}$ 4.$\frac{1}{5}$ 5.0.12 6. $\frac{15}{17}$ 7. $\frac{1}{3}$ 8. 47.5% 9. $\frac{62}{99}$ 10. $\frac{64}{199}$ 11. $\frac{11}{21}$

12. $\frac{2}{9}$ 13. $(i)\frac{12}{17}(ii)\frac{5}{17}$

14.Mean = 2/5, Variance= 144/475, SD= 12/(5√19)

X	0	1	2
P(X)	12/19	32/95	3/95

15. Mean = 4/3, SD = √5/3

X	0	1	2	3
P(X)	5/42	20/42	15/42	2/42

16. (i) 19/144 (ii) $1275/6^4$ (iii) 7/432 (iv) $1125/6^4$

17. The minimum number of trials required is 4.

Answers to Practice Tests

Practice Test 1

2. Not injective 3. {(1,5), (2,11), (8,0)} 4. $f^{-1}(x)=\dfrac{x+4}{3}$ 5. x

6. $(\sqrt{(y+6)}-1)/3$ 8. $(2x-1)/3$ 9. $(7y+4)/(5y-3)$ 10. {1,5,9}

Practice Test 2

1. $5/12$ 2. $7\pi/6$ 3. $7\pi/12$ 4. $\sqrt{3}/2$

5. $7\pi/12$ 6. $\pi/6$ 7. $1/4$ 11. $1/6$

Practice Test 3

1. x = 2 2. B= $\begin{bmatrix}-5 & -4\\ -5 & -6\end{bmatrix}$ 3. λ = 8

2. 4. -256 7. $\begin{bmatrix}\frac{2}{3} & -\frac{1}{6}\\ -1 & \frac{1}{2}\end{bmatrix}$ 10. 300, 100, 200

Practice Test 4

1. $\lambda=-4$ 2. 1 4. $a=\dfrac{1}{2}$

5. $x^{\cos x}\left[\cos x - x\log\sin x + \cos x\log x\right]-\dfrac{4x}{(x^2-1)^2}$

6. $\dfrac{y}{x}\left(\dfrac{x\log y - x}{y\log x - x}\right)$ 7. $\dfrac{1}{\sqrt{1-x^2}}-\dfrac{1}{2\sqrt{x-x^2}}$

8. $\dfrac{-1}{\sqrt{1-x^2}}$ 9. $\dfrac{8b\pi}{3a^2}$ 10. $1-\dfrac{\sqrt{21}}{6}$ 12. 2

SANSKRITI
THE CIVIL SERVICES SCHOOL

Practice Test 5

1. 0.0608 2. $.03x^3 cm^3$ 3. Increasing in $(-\infty,1)\cup\left(1,\frac{8}{5}\right)\cup(2,\infty)$, decreasing in $\left(\frac{8}{5},2\right)$

4. $\frac{y-b\cos^3 t}{x-a\sin^3 t}=\frac{-b}{a}\cot t$ and $\frac{y-b\cos^3 t}{x-a\sin^3 t}=\frac{a}{b}\tan t$

5. $y-\left(2+2\sqrt{3}\right)=2\sqrt{3}(x-2)$ and $y-\left(2-2\sqrt{3}\right)=-2\sqrt{3}(x-2)$

6. Increasing in $(-\infty,1)\cup(1,3)$ and decreasing in $(3,\infty)$

7. local max.at -5 and 0, local min. at -3

8. local max. at $\frac{\pi}{2}$ and local min. at $\frac{\pi}{6}$

9. $\frac{-32}{27\pi}$ 10. $\frac{1}{2\pi} cm/s$

Practice Test 6

$Q1.\ x+2\log\left|\frac{x-1}{x-2}\right|+c$

$Q2.\ \frac{1}{8}\log|1-\sin x|-\frac{1}{8}\log|1+\sin x|-\frac{1}{4\sqrt{2}}\log\left|1-\sqrt{2}\sin x\right|+\frac{1}{4\sqrt{2}}\log\left|1+\sqrt{2}\sin x\right|+c$

$Q3.\ \frac{1}{4}e^{2x}+\frac{1}{8}e^{2x}\left(\sin 2x+\cos 2x\right)+c$ $Q4.\ \frac{-2}{5}\tan^{-1}\frac{x}{2}+\frac{3}{5}\tan^{-1}\frac{x}{3}+c$

$Q6.\ \frac{-1}{3}\left(1+x-x^2\right)^{3/2}+\frac{1}{2}\left[\frac{1}{2}\left(x-\frac{1}{2}\right)\sqrt{1+x-x^2}+\frac{5}{8}\sin^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)\right]+c$

$Q7.\ \frac{1}{5}\log\left|\frac{\tan x/2+2}{\tan x/2-3}\right|+c$ $Q8.\ \pi a$ $Q9.\ \frac{-1}{\sqrt{2}}\log\left(\sqrt{2}-1\right)$ $Q10.\ 3$

$Q11.\ \frac{1}{18}\log|1+\sin x|-\frac{1}{2}\log|1-\sin x|+\frac{4}{9}\log|5-4\sin x|+c$

विद्यया च विमुच्यते SANSKRITI THE CIVIL SERVICES SCHOOL

Practice Test 7

$Q1.\left(x^2-y^2\right)\frac{dy}{dx}-2xy=0$ $Q2.\left(x-y\right)^2\left\{1+\left(\frac{dy}{dx}\right)^2\right\}=\left(x+y\frac{dy}{dx}\right)^2$

$Q3.\ y=\left(x+1\right)\log\left|x+1\right|-x+5$ $Q4.\ cx=ye^{1/x}$

$Q5.\ y=c+a\tan^{-1}\frac{\left(x+y\right)}{a}$ $Q6.\ x^4+6x^2y^2+y^4=8$ $Q7.\ xe^{\tan^{-1}y}=e^{\tan^{-1}y}\left(\tan^{-1}y-1\right)+c$

$Q8.\ x=y^2\left(e^{-1}-e^{-y}\right)$ $Q9.\left(\frac{\sqrt{2}}{6}+\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}\right)sq.units$ $Q10.\frac{11}{6}$ $Q11.\frac{4\sqrt{3}}{3}+\frac{16\pi}{3}\ sq\ units$

Practice Test 8

$Q1. 2\hat{i}-4\hat{j}+4\hat{k}$ $Q3.\ 5x+9y+4z-37=0$ $Q4.(-4,-1,-3)$ $Q5. 15\hat{i}-27\hat{j}+5\hat{k}$

$Q6.\vec{d}=\frac{1}{4}\hat{i}+\frac{1}{4}\hat{j}+\frac{3}{4}\hat{k}$ $Q8.(1,-2,7)$ $Q9.\vec{r}.\left(33\hat{i}+45\hat{j}+50\hat{k}\right)-41=0$ $Q10.$ distance $=13$

$Q11.$ Foot of perpendicular is $(1,3,0)$, perpendicular distance $=\sqrt{6}$, image is $(-1,4,1)$

Practice Test 9

$Q1.\frac{1}{3}$ $Q2.(i)\frac{11}{12}$ $(ii)\frac{5}{12}$ $(iii)\frac{7}{8}$ $Q3.\frac{307}{686}$ $Q4.\frac{5}{11}$ $Q5.\frac{18}{143}$ $Q6.\frac{3}{13}$ $Q7.\frac{13}{50}$ $Q8.$ *no.* of trials ≥ 4

$Q9.$

X	0	1	2	3
P(X)	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

$Q10.$

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$Mean=\frac{6}{5}$, Variance $=\frac{14}{25}$ $Q11.\frac{2}{5}$